CH 4 BOATS AND STREAMS

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (b) Downstream speed = 15 + 5 = 20 km/h.

$\therefore$ Required distance $= 20 \times \frac{24}{60} = 8$km.

2. (d) Let x be the speed of the boat.

and y the speed of the current.

$\therefore \quad \frac{20}{x-y} + \frac{20}{x+y} = \frac{25}{36}$

In this equation there are two variables, but only one equation, so, the value of 'x' cannot be determined.

3. (c) Here downstream speed = 15 km/hr and upstream speed = 5 km/hr

$\therefore$ Speed of the boat

$= \frac{15+5}{2}$ = 10 km/h

4. (c) Let the rate of stream = S km/h

Given 20 + S = 25

$\Rightarrow$ S = 5 km/h

5. (a) Let the rate against the current be x km/hr. Then,

$\frac{12-x}{2} = 1.5 \Rightarrow 12 - x = 3 \Rightarrow x = 9$ km / hr

6. (a) Man's speed in upstream = 4 – 2 = 2 km/h.

$\therefore$ Required time $= \frac{6}{2} = 3$ km / h

7. (a) Let the speed of rowing be X. Then the equation formed is $\frac{9}{X-2} + \frac{9}{X+2} = 6$.

On solving, we get the value of X as 4.

8. (c) Speed with the stream = 10.56 km. an hour.

Speed against the stream

$= \frac{352}{4} \times \frac{60}{1000} = 5.28$ km an hour

$\Rightarrow$ Speed in still water

$= \frac{1}{2}(10.56 + 5.28) = 7.92$ kmph

9. (d) Required distance between A and B

$= \frac{3\left((9)^2 - (3)^2\right)}{2(9)} = \frac{3(81-9)}{18}$

$= \frac{72}{6} = 12$ km.

10. (a) If the rate of the stream is x, then

2(4.5 – x) = 4.5 + x

$\Rightarrow$ 9 – 2x = 4.5 + x

$\Rightarrow$ 3x = 4.5

$\Rightarrow$ x = 1.5 km/hr

11. (d) Given, time taken by the boat to place and back = 3 hrs. and given speed of boat in still water = 4 km/hr

There is no speed of the stream given and also no down stream and upstream speed is given.

$\therefore$ Can't calculate the distance.

12. (d) Given speed of boat in water = 4 km/hr = x (say)

and speed of current = 2 km/hr = y (say)

As we know, Distance = Speed × Time

$\therefore$ Upstream speed = (x – y) km/hr

and time = 9 hr (given)

$\therefore$ distance upstream = (x – y). 9

and downstream speed = (x + y) km/hr

Now, distance downstream = distance upstream (given)

$\therefore$ (x– y) 9 = (x + y). T

$\Rightarrow T = \left(\frac{x-y}{x+y}\right)9 = \left(\frac{4-2}{4+2}\right)9 = 3\text{hrs.}$

13. (b) Rate downstream = $\left(\frac{16}{2}\right)$ kmph = 8 kmph;

Rate upstream = $\left(\frac{16}{4}\right)$ kmph = 4 kmph.

$\therefore$ Speed in still water = $\frac{1}{2}$ (8 + 4) = 6 km/h.

14. (a) Man's upstream speed $= \frac{36}{6} = 6\text{ km/hr}$

Speed of stream = 8 – 6 = 2 km/h

$\therefore$ Man's downstream speed = 8 + 2 = 10 km/h

Hence, required distance = 10 × 10 = 100 km

15. (d) Man's speed in still water = 4.5 km/h

Let speed of stream = S km/h

Here, $T_u = 2T_d$

$\therefore \frac{\text{Distance}}{4.5-S} = 2\left(\frac{\text{Distance}}{S+4.5}\right)$

$\Rightarrow 4.5 + S = 9 - 2S$

$\Rightarrow 3S = 4.5 \Rightarrow S = 1.5$ km/h

16. (c) Let the distance travelled during both upward and downward journey be x km.

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}}$

$= \frac{x+x}{\frac{x}{16}+\frac{x}{28}} = \frac{2}{\frac{28+16}{28\times16}}$

$= \frac{2\times28\times16}{44} = 20.36\text{ km/h}$

17. (c) Relative speed of the boats = 15 km/ hour

$= \frac{15}{60} = \frac{1}{4}$ km/min

i.e., they cover $\left(\frac{1}{4}\right)$ km in the last one minute before collision

18. (a) If X be the speed of man in still water, Y the speed of stream, then Y = 2.

$X - 2 = \frac{9}{3}$ or X = 5.

Now, X + 2 = 7, hence time required

= 9/7 hours..

EXERCISE 2

1. (d) Speed of the boat in still water = 10 mph

Let the speed of the stream = x mph

Then, speed of boat with downward stream

= (10 + x) mph

Speed of boat with upward stream

= (10 – x) mph

Now, $\frac{36}{(10+x)} + \frac{90}{60} = \frac{36}{(10-x)}$

or $\frac{1}{4} = 6\left(\frac{1}{10-x} - \frac{1}{10+x}\right)$

or $\frac{1}{4} = 6\left(\frac{2x}{100-x^2}\right)$

or $100 - x^2 = 48x$

or $x^2 + 48x - 100 = 0$

or $x = 2$ mph $[x \neq -50]$

2. (b) Let the speed of the boat in still water be x km/hr

Speed of the stream = 2 km/ hr

$\therefore$ Speed of the boat downstream

= (x + 2) km/hr

Speed of the boat upstream

= (x – 2) km/hr

$\therefore \frac{8}{x+2} + \frac{8}{x-2} = 1\frac{2}{3} = \frac{5}{3}$

$\Rightarrow 24x - 48 + 24x + 48 = 5(x^2 - 4)$

$\Rightarrow 5x^2 - 48x - 20 = 0$

$\Rightarrow x = \frac{48 \pm \sqrt{2304+400}}{10}$

$= \frac{48 \pm 52}{10} = 10, -0.4$

$\therefore$ Speed of the boat in still water = 10 km/hr.

3. (c) Let x be the speed of boat in still water and y be the speed in current.

$\therefore$ Speed of the boat downstream

= (x + y) km/hr and speed of the boat upstream = (x – y) km/hr.

According to the question :

$2(x + y) = 3(x - y)$

$\Rightarrow 2x + 2y = 3x - 3y \Rightarrow 5y = x$

$\Rightarrow \frac{5}{1} = \frac{x}{y}$

Hence, the ratio of speed in still water to speed in current is 5:1

4. (c) Let the speed of man in still water be v_m and the speed of stream be v_s. Then

$$(v_m - v_s) \times \left(\frac{45}{4 \times 60}\text{hr}\right) = \frac{3}{4}\text{km} \quad ....(1)$$

Also, $$(v_m + v_s) \times \left(\frac{29}{4 \times 60}\text{hr}\right) = \frac{3}{4}\text{km} \quad ....(2)$$

Now, we solve for v_m.

(1) $\Rightarrow v_m - v_s$

$$= \frac{3}{4} \times \frac{4 \times 60}{45} = \frac{3 \times 60}{45} = 4$$

and (2) $\Rightarrow v_m + v_s$

$$= \frac{3}{4} \times \frac{4 \times 60}{29} = \frac{3 \times 60}{29} = \frac{180}{29}$$

By adding (1) and (2), we get

$$2v_m = 4 + \frac{180}{29} \Rightarrow v_m = 5$$

Hence, the speed of the man in still water

= 5 km/hr.

5. (c) The speed against the current of the stream

= 2 km (in 1 hr)

The speed along the current of the stream

= 1km (in 10 min)

$= \frac{1 \times 60}{10}$ (in 1 hr)

= 6 km (in 1 hr)

$\therefore$ Speed in stationary water = 6 – 2 = 4 km

Thus, speed in stationary water = 4km and distance = 5 km

As, we know, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ Time taken to cover the distance 5 km

$= \frac{5}{4}\text{hour} = 75$ min

= 60 min + 15 min = 1 hour 15 minutes

6. (d) Let v_m = velocity of man = 48 m/min

Let v_c = velocity of current

then t_1= time taken to travel 200 m against the current.

i.e., $$t_1 = \frac{200}{v_m - v_c} \quad ....(1)$$

and t_2 time taken to travel 200 m with the current

i.e., $$t_2 = \frac{200}{v_m + v_c} \quad ....(2)$$

Given : $t_1 - t_2 = 10$ min

$$\therefore \frac{200}{v_m - v_c} - \frac{200}{v_m + v_c} = 10$$

$$\Rightarrow v_m^2 - v_c^2 = 40v_c$$

$$\Rightarrow v_c^2 + 40v_c - (48)^2 = 0$$

$$\Rightarrow v_c = 32, -72$$

Hence, speed of the current

$= 32 \ (\because v_c \neq -72)$.

7. (a) Here, Distance for downstream

= 2(Distance for upstream)

Let speed of stream = S km/h.

$\therefore$ 4 + S = 2(4 – S)

$\Rightarrow$ $S = \frac{4}{3} = 1.3 km/h$.

8. (d) Let the speed of the stream be x km/h.

Then, upstream speed = (15 – x) km/h.

and downstream speed = (15 + x) km/h.

Now, $\frac{30}{(15+x)} + \frac{30}{(15-x)} = 4.5$

Checking with options, we find that x = 5 km/h.

9. (b) Man's speed in downstream = $\frac{60}{6} = 10 km/hr$.

$\therefore$ Man's speed in still water = 10 – 3 = 7 km/h

Man's speed in upstream = 7 – 3 = 4 km/h

$\therefore$ Required time = $\frac{16}{4}$ = 4 hrs

10. (a) Suppose he moves 4 km downstream in x hours. Then,

Downstream speed $= \frac{4}{x} km/hr$

Upstream speed $= \frac{3}{x} km/hr$

$\therefore \frac{48}{4/x} + \frac{48}{3/x} = 14 \Rightarrow x = \frac{1}{2} hr$

$\therefore$ Downstream speed = 8 km/h

and upstream speed = 6 km/h

Rate of the stream $= \frac{1}{2}(8-6) = 1 km/hr$

11. (b) Time taken by the boat during downstream journey $= \frac{50}{60} = \frac{5}{6} h$

Time taken by the boat in upstream journey

$= \frac{5}{4} h$

Average speed

$= \frac{2 \times 50}{\frac{5}{6} \times \frac{5}{4}} = \frac{100 \times 24}{50} = 48$ mph

12. (a) Clearly, Ram moves both ways at a speed of 12 km/h. So, average speed of Ram = 12 km/h.

Shyam moves downstream at the speed of (10 + 4)

= 14 km/h

and upstream at the speed of (10 – 4) = 6 km/h.

So, average speed of Shyam

$= \left(\frac{2 \times 14 \times 6}{14+6}\right)$ km/h

$= \frac{42}{5}$ km/h = 8.4 km/h.

Since the average speed of Ram is greater, he will return to A first.

13. (a) Let speed of the boat in still water be x km/h and speed of the current be y km/h.

Then, upstream speed = (x – y) km/h

and downstream speed = (x + y) km/h

Now, $\frac{24}{(x-y)} + \frac{28}{(x+y)} = 6$ ...(i)

and $\frac{30}{(x-y)} + \frac{21}{(x+y)} = \frac{13}{2}$ ...(ii)

Solving (i) and (ii), we have

x = 10 km/h and y = 4 km/h

14. (d) Let the speed of the boat in still water be S kmph. Then,

Downstream speed = (S + 3) kmph,

Upstream speed = (S – 3) kmph.

$\therefore (S+3) \times 1 = (S-3) \times \frac{3}{2}$

$\Rightarrow$ 2S + 6 = 3S – 9

$\Rightarrow$ S = 15 kmph.

15. (b) Downstream speed = (14 + 4) km/h = 18 km/h.

Upstream speed = (14 – 4) km/h = 10 km/h.

Let the distance between A and B be d km. Then,

$\frac{d}{18} + \frac{(d/2)}{10} = 19 \Rightarrow \frac{d}{18} + \frac{d}{20} = 19 \Rightarrow \frac{19d}{180} = 19$

$\Rightarrow$ d = 180 km.

∴ Remaining portion to be filled by pipe B

$= \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30}$

∴ Time taken by pipe B to fill $\frac{1}{30}$ of the cistern

$= \frac{1}{30} \times 30 = 1\text{min}$

Hence, total time = (55 × 3) + 1 + 1 = 167 min.

9. (a) Let cistern will be full in x min. Then,
part filled by B in x min + part filled by A in (x – 4) min = 1

$\Rightarrow \frac{x}{16} + \frac{x-4}{12} = 1$

$\Rightarrow x = \frac{64}{7} = 9\frac{1}{7}$ hours.

10. (a) Let A was turned off after x min. Then,
cistern filled by A in x min + cistern filled by B in (x + 23) min = 1

$\Rightarrow \frac{x}{45} + \frac{x+23}{40} = 1$

$\Rightarrow 17x + 207 = 360 \Rightarrow x = 9\text{min}$.

11. (a) Let cistern will be full in x min. Then,
part filled by A in x min + part filled by B in (x – 1) min + part filled by C in (x – 2)min = 1

$\Rightarrow \frac{x}{3} + \frac{x-1}{4} + \frac{x-2}{6} = 1$

$\Rightarrow 9x = 19 \Rightarrow x = \frac{19}{9} = 2\frac{1}{9}\text{min}$

12. (b) Total number of pipes = 6 (given)
Let number of inlet pipes = x
∴ number of outlet pipes = 6 –x
Now, Inlet pipe fill the tank in 9 hours and outlet pipe empty it in 6 hours.

∴ Total part filled in 1 hour = $\frac{x}{9} - \frac{6-x}{6}$

When all the pipes are opened.
But given total part filled in 9 hr

∴ $\frac{x}{9} - \frac{6-x}{6} = \frac{1}{9} \Rightarrow 6x - 54 + 9x = \frac{54}{9} = 6$

⇒ 15x = 60 ⇒ x = 4

Hence, number of inlet pipes = 4.

13. (a) Part filled in 10 hours

$= 10\left(\frac{1}{15} + \frac{1}{20} - \frac{1}{25}\right) = \frac{23}{30}$.

Remaining part = $\left(1 - \frac{23}{30}\right) = \frac{7}{30}$.

(A + B)'s 1 hour's work = $\left(\frac{1}{15} + \frac{1}{20}\right) = \frac{7}{60}$.

$\frac{7}{60} : \frac{7}{30} :: 1 : x$

or $x = \left(\frac{7}{30} \times 1 \times \frac{60}{7}\right) = 2$ hours.

∴ The tank will be full in (10 + 2) hrs = 12 hrs.

14. (c) (A + B)'s 1 hour's work

$= \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{9}{60} = \frac{3}{20}$

(A + C)'s 1 hour's work

$= \left(\frac{1}{12} + \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$

Part filled in 2 hrs

$= \left(\frac{3}{20} + \frac{2}{15}\right) = \frac{17}{60}$

Part filled in 6 hrs

$= \left(3 \times \frac{17}{60}\right) = \frac{17}{20}$

Remaining part

$= \left(1 - \frac{17}{20}\right) = \frac{3}{20}$

Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.

$$\Rightarrow\left(S_u \times 8\frac{4}{5}\right) = (S_d \times 4) \Rightarrow \frac{44}{5}S_u = 4S_d \Rightarrow S_d = \frac{11}{5}S_u.$$

∴ Required ratio

$$= \left(\frac{S_d + S_u}{2}\right) : \left(\frac{S_d - S_u}{2}\right) = \left(\frac{16S_u}{5} \times \frac{1}{2}\right) : \left(\frac{6S_u}{5} \times \frac{1}{2}\right)$$

$$= \frac{8}{5} : \frac{3}{5} = 8 : 3.$$

3. (c) Let upstream speed = S_u kmph and downstream speed = S_d kmph.

Then, $\frac{24}{S_u} + \frac{36}{S_d} = 36$ ...(1)

and $\frac{36}{S_u} + \frac{24}{S_d} = \frac{13}{2}$ ...(2)

Adding (1) and (2), we get :

$$60\left(\frac{1}{S_u} + \frac{1}{S_d}\right) = \frac{25}{2} \text{ or } \frac{1}{S_u} + \frac{1}{S_d} = \frac{5}{24} \quad ....(3)$$

Subtracting (1) and (2), we get :

$$12\left(\frac{1}{S_u} - \frac{1}{S_d}\right) = \frac{1}{2} \text{ or } \frac{1}{S_u} - \frac{1}{S_d} = \frac{1}{24} \quad ....(4)$$

Adding (3) and (4), we get :

$$\frac{2}{S_u} = \frac{6}{24} \text{ or } S_u = 8.$$

So, $\frac{1}{8} + \frac{1}{S_d} = \frac{5}{24} \Rightarrow \frac{1}{S_d} = \left(\frac{5}{24} - \frac{1}{8}\right) = \frac{1}{12}$

⇒ $S_d = 12.$

∴ Speed upstream = 8 kmph,

Speed downstream = 12 kmph.

Hence, rate of current

$$= \frac{1}{2}(12 - 8) \text{ kmph} = 2 \text{ kmph}.$$

4. (b) Let man's speed be a km/hr.

Let stream's speed be b km/hr.

a + b = 3, a – b = 2, 2a = 5,

a = 5/2 = 2.5 km/hr.

To swim 7 km, time required

$$= \frac{7}{2.5} = 2.8 \text{ hours}.$$

(Must do mentally in 20 sec. max.)

5. (c) Speed of speed-boat = 16 – 3 = 13 km/hr.

∴ Speed of boat against the current

= 13 – 3 = 10 km/hr.

$$= \frac{1}{2}\left[\begin{array}{l}\text{Speed against the stream} \\ -\text{ speed with the stream}\end{array}\right]$$

$$= \frac{1}{2}[8 - 6] = 1 \text{ km/h}.$$

6. (a) Let the speed of the boatman be x km/hr and that of stream by y km/hr. Then

$$\frac{12}{x+y} = \frac{4}{x-y}$$

⇒ 12x – 12y = 4x + 4y

⇒ 8x = 16y

⇒ x = 2y

Now $\frac{45}{x+y} + \frac{45}{x-y} = 20$

⇒ 45 + 135 = 60 y ⇒ 180 = 60y

⇒ y = 3km/hr.

7. (b) Let the speed of the stream be x km/hr and distance travelled be S km. Then,

$$\frac{S}{12+x} = 6 \text{ and } \frac{S}{12-x} = 9$$

$$\Rightarrow \frac{12-x}{12+x} = \frac{6}{9}$$

⇒ 108 – 9x = 72 + 6x

⇒ 15x = 36

$$\Rightarrow x = \frac{36}{15} = 2.4 \text{ km / hr}.$$

8. (d) Total distance covered = 2 × 91 km = 182 km

Time taken = 20 hours

$\therefore$ Average speed = $\frac{182}{20} = 9.1 \text{ km/h}$

Let the speed of flow of the river = x km/hr

then, $\frac{10^2 - x^2}{10} = 9.1 \Rightarrow 100 - 91 = x^2 \Rightarrow x = \pm 3$

Hence, rate of flow of the river = 3 km/h

9 (a) Downstream speed = x + y = 6 km/hr.

Upsteam speed = x – y = 4 km/hr.

$\therefore$ Speed in still water

$= \frac{x+y+x-y}{2} = \frac{6+4}{2} = 5$

= 5 km/hr.

and speed of the current

$= \frac{x+y-x+y}{2} = \frac{6-4}{2} = 1 \text{km/hr}$

10. (c) Let x be the speed of the swimmer and y be the speed of the current.

Let d be the distance.

$\therefore$ By question $\frac{d}{x+y} = 5\text{hr}$ and $\frac{d}{x-y} = 7\text{hr}$.

$\Rightarrow$ d = 5x + 5y and d = 7x – 7y

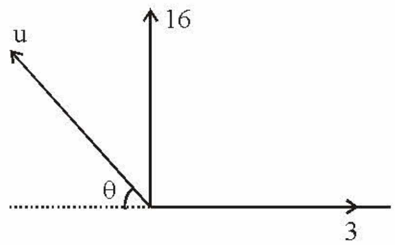
$\Rightarrow$ 5x + 5y = 7x – 7y

$\Rightarrow$ 12 y = 2x $\Rightarrow$ x = 6y.

But given, y = 1 km/hr

$\therefore$ x = 6 km/hr.

11. (d)



Let the speed of the boat be u km per hour.

$\therefore$ u cos θ = 3, u sin θ = 16

$\Rightarrow \tan\theta = \frac{16}{3}$

$\Rightarrow \sin\theta = \frac{16}{\sqrt{265}}$

Since, u sin θ = 16

$\Rightarrow u.\frac{16}{\sqrt{265}} = 16$

$\Rightarrow u = \sqrt{265} =$ 16.28 km per hour

$\therefore$ Speed of the boat against the current

= u – 3 = 16.28 – 3 = 13.28 km per hour.

12. (a) Man's speed in still water = 4 km/h.

Speed of stream = 2 km /h

S_d = 4 + 2 = 6 km/h

S_u = 4 – 2 = 2 km/h

Let distance be d km. Then,

$T_d = \frac{d}{6}$ and $T_u = \frac{d}{2}$

$T_d + T_u = \frac{d}{6} + \frac{d}{2}$

$\Rightarrow 6 = \frac{4d}{6}$

($\because$ Total time $T_d + T_u$ = 6 hrs)

$\Rightarrow$ d = 9 km.

13. (d) Speed of the boat in still water = 10 mph

Let the speed of the stream = x mph

Then, speed of boat with downward stream

= (10 + x) mph

Speed of boat with upward stream = (10 – x) mph

Now, $\frac{36}{(10+x)} + \frac{90}{60} = \frac{36}{(10-x)}$

or $\frac{1}{4} = 6\left(\frac{1}{10-x} - \frac{1}{10+x}\right)$

or $\frac{1}{4} = 6\left(\frac{2x}{100-x^2}\right)$

or $100 - x^2 = 48x$

or $x^2 + 48x - 100 = 0$

or x = 2 $[x \neq -50]$