CH 5 GEOMETRY

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (a) a + 36° + 70° = 180° (sum of angles of triangle)

$\Rightarrow$ a = 180° – 36° – 70° = 74°

b = 36° + 70°(Ext. angle of triangle) = 106°

c = a – 50° (Ext. angle of triangle)

=74° – 50° = 24°.

2. (b) Since the sum of all the angle of a quadrilateral is 360°

We have $\angle$ ABC + $\angle$ BQE + $\angle$ DEF + $\angle$ EPB = 360°

$\therefore \angle$ ABC + $\angle$ DEF = 180°

[$\because$ BPE = EQB = 90°]

3. (b) m $\angle$ AHG = 180 – 108 = 72^0

$\therefore$ $\angle$ AHG = $\angle$ ABC .....(same angle with different names)

$\therefore$ Δ AHG – ΔABC .....(AA test for similarity)

$\frac{AH}{AB} = \frac{AG}{AC}$; $\frac{6}{12} = \frac{9}{AC}$

$\therefore AC = \frac{12 \times 9}{6} = 18$

$\therefore$ HC = AC – AH = 18 – 6 = 12

4. (b) In $\Delta ABC, \angle C = 180 - 90 - 30 = 60°$

$\therefore \Delta DCE = \frac{60}{2} = 30°$

Again in ΔDEC, $\Delta CED = 180 - 90 - 30 = 60°$

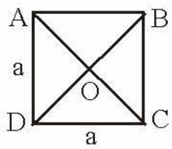
5. (c) In a right angled Δ, the length of the median is $\frac{1}{2}$ the length of the hypotenuse . Hence $BD = \frac{1}{2}AC = 3cm.$

6. (a) $\angle D = 180 - \angle B = 180 - 70 = 110°$

$\therefore \angle ACD = 180 - \angle D - \angle CAD$

$180 - 110 - 30 = 40°$

7. (b)



ABCD is square $a^2 = 4 \Rightarrow a = 2$

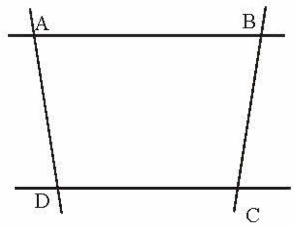
$ac = BD = 2\sqrt{2}$

perimeters of four triangles

$= AB + BC + CD + DA + 2(AC + BD)$

$= 8 + 2(2\sqrt{2} + 2\sqrt{2}) = 8(1 + \sqrt{2})$

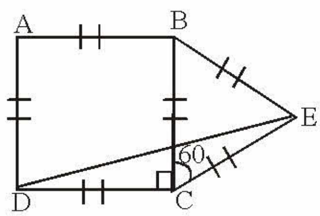
8. (d) The quadrilateral obtained will always be a trapeziam as it has two lines which are always parallel to each other.



9. (b) It is a rectangle.

(In a cyclic parallelogram each angle is equal to 90°. So, it is definitely either a square or a rectangle. Since the given cyclic parallelogram has unequal adjacent sides, it is a square.)

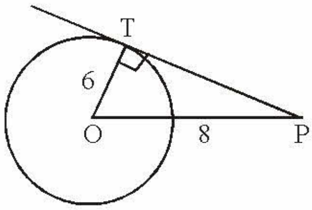
10. (a)



In $\Delta DEC, \angle DCE = 90° + 60° = 150°$

$$\angle CDE = \angle DEC = \frac{180-150}{2} = 15°$$

11. (e) OP = 8 cm, OT = 6 cm



$\therefore PT = \sqrt{OP^2 - OT^2} = \sqrt{8^2 - 6^2} = \sqrt{28}.$

12. (d) $\angle MBA = 180^0 - 95^0 = 85^0$

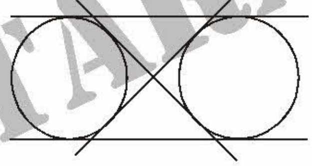
$\angle AMB = \angle TMN$ ...(Same angles with different names)

$\therefore \Delta MBA - \Delta MNT$ ......(AA test for similarity)

$$\frac{MB}{MN} = \frac{AB}{NT} \quad \text{.......(proportional sides)}$$

$$\frac{10}{MN} = \frac{5}{9} \qquad \therefore MN = \frac{90}{5} = 18.$$

13. (a) Four tangents can be drawn to two non-intersecting circles in the following manner :



14. (c) Tangent at any point of a circle is $\perp$ to the radius

In ΔOPT, $OP^2 = PT^2 + OT^2$

$(13)^2 = (12)^2 + OT^2$

$\Rightarrow 169 - 144 = OT^2$

$\Rightarrow 25 = OT^2 \Rightarrow 5 = OT$

15. (c) Let the angles of the triangle be 5x, 3x and 2x.

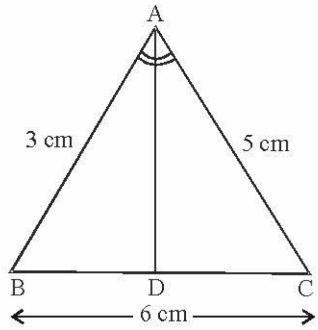
Now, 5x + 3x + 2x = 180°

or 10x = 180 or x = 18

or Angles are 36, 54 and 90°

Given Δ is right angled.

16. (b)



As AD bisects $\angle BAC$, we have

$$\frac{BD}{AB} = \frac{DC}{AC} \quad \text{or} \quad \frac{DC}{BD} = \frac{5}{3}$$

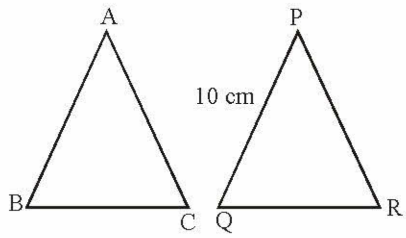
or $\frac{DC}{BD} + 1 = \frac{5}{3} + 1$

or $\frac{DC + BD}{BD} = \frac{5+3}{3}$

or $\frac{BC}{BD} = \frac{8}{3}$

or $BD = \frac{BC \times 3}{8} = \frac{6 \times 3}{8} = \frac{9}{4} = 2.25$ cm

17. (d)



ΔABC and ΔPQR are similar.

$$\frac{AB}{PQ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{36}{24}$$

or $AB = \frac{36}{24} \times 10 = 15$

18. (c) We have,

$\angle OBC = \angle OCB = 37°$

(equal angles of an isosceles triangle)

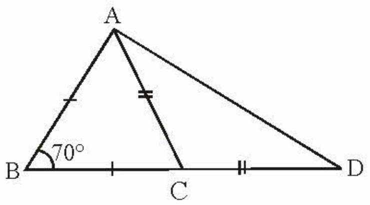
$\Rightarrow \angle COB = 180° - (37° + 37°) = 106°$

Therefore, $\angle BAC$

$$= \frac{1}{2}\angle COB = \frac{106°}{2} = 53°$$

19. (b) $\angle ACB = \angle BAC$

(Angles opposite equal sides are equal)



Similarly, $\angle ADC = \angle CAD$

$\therefore \angle ACB = \angle BAC$

$$= \left(\frac{180° - 70°}{2}\right) = 55°$$

$\Rightarrow \angle ADC = \angle CAD$

$$= \frac{180° - 125°}{2} = 27.5°$$

20. (b) The sum of the interior angles of a polygon of n sides is given by the expression $(2n - 4)\ \frac{\pi}{2}$

$$\Rightarrow (2n-4)\times\frac{\pi}{2} = 1620\times\frac{\pi}{180}$$

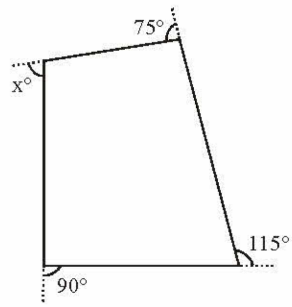
$$(2n-4) = \frac{1620\times 2}{180} = 18$$

or $2n = 22$ or $n = 11$

Thus the no. of sides of the polygon are 11.

EXERCISE 2

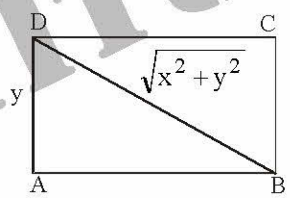
1. (c) Sum of all the interior angles of a polygon taken in order is 360°.



i.e. $x + 90 + 115 + 75 = 360$

or $x = 360° - 280° = 80°$ or $x = 80°$

2. (d)



According to question,

$$(x+y) - \sqrt{x^2+y^2} = \frac{x}{2}$$

$$(x+y) - \frac{x}{2} = \sqrt{x^2+y^2}$$

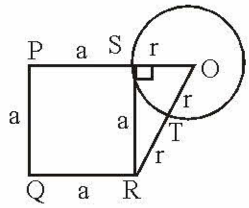
$$\left(\frac{x}{2}+y\right)^2 = x^2+y^2$$

$$\frac{x^2}{4}+y^2+xy = x^2+y^2$$

$$x^2+4xy = 4x^2$$

$$4xy = 3x^2 \Rightarrow 4y = 3x \Rightarrow \frac{y}{x} = \frac{3}{4}$$

3. (a)

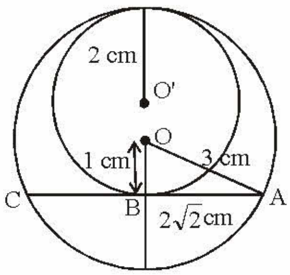


In $\Delta SOR, a^2 + r^2 = (2r)^2 = 4r^2$

$\Rightarrow a^2 = 3r^2$ or $a = \sqrt{3}r$

$\therefore \dfrac{\text{Area of circle}}{\text{Area of square}} = \dfrac{\frac{22}{7}\times r^2}{(\sqrt{3}r)^2} = \dfrac{22}{7\times 3} = \dfrac{22}{21} = \dfrac{\pi}{3}$

4. (d)



$AB = \sqrt{3^2 - 1^2} = 2\sqrt{2}$ cm

$\therefore$ AC = $4\sqrt{2}$ cm

5. (c) Given AB is a circle and BT is a tangent,

$\angle BAO = 32^o$

Here, $\angle OBT = 90^o$

[$\because$ Tangent is $\perp$ to the radius at the point of contact]

OA = OB [Radii of the same circle]

$\therefore \angle OBA = \angle OAB = 32^o$

[Angles opposite to equal side are equal]

$\therefore \angle OBT = \angle OBA + \angle ABT = 90^o$

or $32^o + x = 90^o$.

$\angle x = 90^o - 32^o = 58^o$.

Also, $\angle AOB = 180^o - \angle OAB - \angle OBA$

$= 180^o - 32^o - 32^o$

$= 116^o$

Now $Y = \dfrac{1}{2}$ AOB [Angle formed at the center of a circle is double the angle formed in the remaining part of the circle]

$= \dfrac{1}{2} \times 116^o = 58^o$.

6. (a) In ΔABC, DE | | BC

By applying basic Proportionality theorem,

$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

But $\dfrac{AD}{DB} = \dfrac{3}{5}$ (Given)

$\therefore \dfrac{AE}{EC} = \dfrac{3}{5}$ or $\dfrac{AE}{EC+AE} = \dfrac{3}{5+3}$ or $\dfrac{AE}{AC} = \dfrac{3}{8}$

or $\dfrac{AE}{5.6} = \dfrac{3}{8} \Rightarrow 8AE = 3 \times 5.6 \Rightarrow AE = 3 \times 5.6$ /8

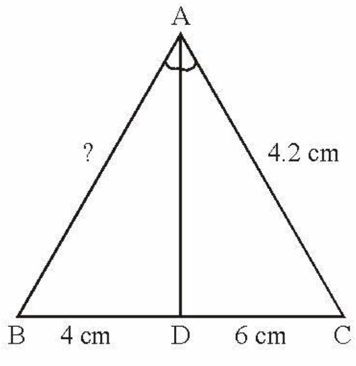
$\therefore$ AE = 2.1 cm.

7. (b) $\Delta PAB \sim \Delta PQR$

$\dfrac{PB}{AB} = \dfrac{PR}{QR} \Rightarrow \dfrac{PB}{3} = \dfrac{6}{9}$

$\therefore$ PB = 2 cm

8. (a)

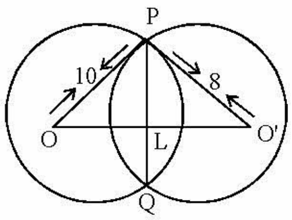


$\Delta ABD \sim \Delta ACD$

$\dfrac{AC}{DC} = \dfrac{AB}{BD} \Rightarrow \dfrac{4.2}{6} = \dfrac{AB}{4}$

$\therefore$ AB = 2.8 cm

9. (b) Here, OP = 10 cm; O'P = 8 cm



PQ = 12 cm

$\therefore$ PL = 1/2 PQ $\Rightarrow PL = \frac{1}{2}\times 12 \Rightarrow PL = 6\,cm$

In rt. $\Delta OLP, OP^2 = OL^2 + LP^2$
(using Pythagoras theorem)

$\Rightarrow (10)^2 = OL^2 + (6)^2 \Rightarrow OL^2 = 64$; OL = 8

In $\Delta O'LP, (O'L)^2 = O'P^2 - LP^2 = 64 - 36 = 28$

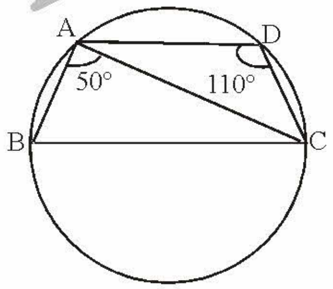
$O'L^2 = 28 \Rightarrow O'L = \sqrt{28}$

O'L = 5.29 cm

$\therefore OO' = OL + O'L = 8 + 5.29$

OO' = 13.29 cm

10. (a) $\angle ABC + \angle ADC = 180°$ (sum of opposites angles of cyclic quadrilateral is 180°)



$\Rightarrow \angle ABC + 110° = 180°$

(ABCD is a cyclic quadrilateral)

$\Rightarrow \angle ABC = 180 - 110 \Rightarrow \angle ABC = 70°$

$\because$ AD || BC

$\therefore \angle ABC + \angle BAD = 180°$ (Sum of the interior angles on the same side of transversal is 180°)

$70° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 180° - 70° = 110°$

$\Rightarrow \angle BAC + \angle DAC = 110°$

$\Rightarrow 50° + \angle DAC = 110°$

$\Rightarrow \angle DAC = 110° - 50° = 60°$

11. (c) $b = \frac{1}{2}(48°)$

($\angle$at centre = 2 at circumference on same PQ) 24°

$\angle$ AQB = 90° ($\angle$In semi- circle)

$\angle$ QXB = 180° – 90° – 24°

($\angle$sum of Δ) = 66°

12. (a) $AO = \sqrt{OQ^2 - AQ^2} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$

Now, from similar Δs QAO and QPR

OR = 2OA = 2× 3 = 6 cm.

13. (b) m $\angle$ ACD = $\frac{1}{2}$ M(are CXD) = m $\angle$ DEC

$\therefore$ m $\angle$ DEC = x = 40°

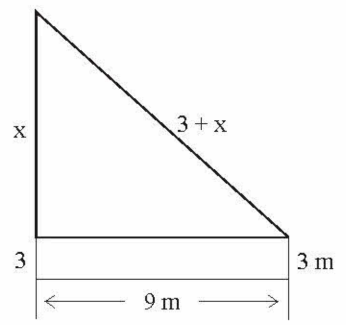
$\therefore$ m $\angle$ ECB = $\frac{1}{2}$ m (are EYC) = m $\angle$ EDC

$\therefore$ m $\angle$ ECB = y = 54^0

54 + x + z = 180° ....(Sum of all the angles of a triangle)

54 + 40 + z = 180° $\therefore$ z = 86°.

14. (b)

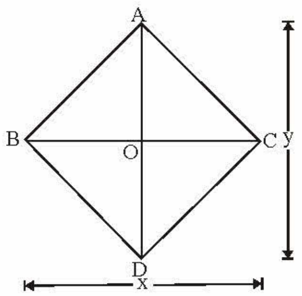


Using pythagoras , $x^2 + 81 = (3 + x)^2$

or $x^2 + 81 = 9 + x^2 + 6x \Rightarrow 6x = 72$ or $x = 12m$

Height of wall = 12 + 3 =15 m

15. (a) Let the diagonals of the rhombus be x and y and the its sides be x.



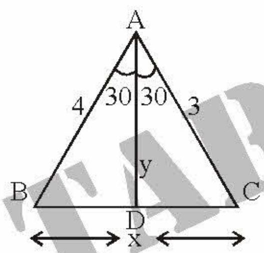
Now, $x^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2$

or $x^2 - \frac{x^2}{4} = \frac{y^2}{4}$

$3x^2 = y^2$

or $\frac{x}{y} = \frac{1}{\sqrt{3}}$ or $y : x = \sqrt{3} : 1$

16. (b)



Using the theorem of angle of bisector,

$\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3} \Rightarrow BD = \frac{4}{7}x \ \& DC = \frac{3}{7}x$

In ΔABD, by sine rule, $\frac{\sin 30}{4/7x} = \frac{\sin B}{y}$ ....(1)

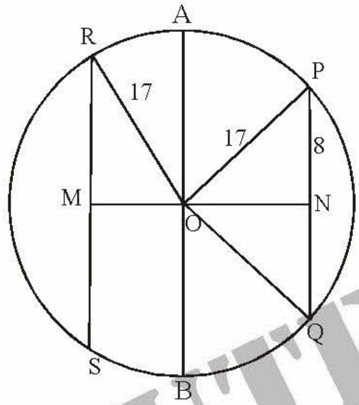
In ΔABC, by sine rule; $\frac{\sin 60}{x} = \frac{\sin B}{3}$

or $\frac{\sqrt{3}}{2x} = \frac{\sin 30.y}{4/7x \times 3}$

[putting the value of sin B from (1)]

$\Rightarrow y = \frac{\sqrt{3}}{2x} \times \frac{4}{7} x \times 3 \times \frac{2}{1} = \frac{12\sqrt{3}}{7}$

17. (c) Let PQ and RS be two parallel chords of the circle on the opposite sides of the diameter AB and PQ = 16 cm



Now, PN = 8 (Since ON is the perpendicular bisector)

In ΔPON,

$ON^2 = OP^2 - PN^2$

$= (17)^2 - (8)^2 = 289 - 64 = 225$

or $ON = 15$

$\Rightarrow \therefore OM = 23 - 15 = 8$

In ΔORM,

$RM^2 = OR^2 - OM^2$

$= 17^2 - 8^2 = 289 - 64 = 225$

or $RM = 15$

$\Rightarrow RS = 15 \times 2 = 30$ cm

18. (c) Let n be the number of sides of the polygon

Now, sum of interior angles = 8 × sum of exterior angles

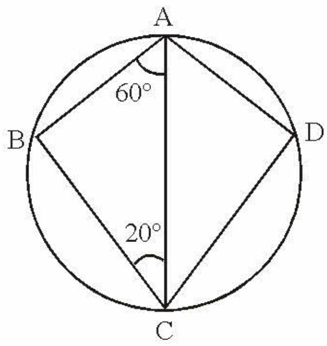
i.e. $(2n - 4) \times \frac{\pi}{2} = 8 \times 2\pi$

or $(2n - 4) = 32$

or $n = 18$

19. (c) In ΔABC,

$\angle B = 180° - (60° + 20°)$ (By ASP)



$\Rightarrow \angle B = 100°$

But $\angle B + \angle D = 180°$

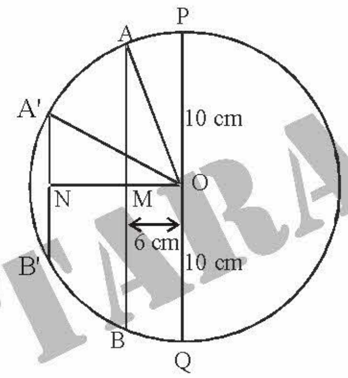
($\because$ ABCD is a cyclic quadrilateral;

Sum of opposite is 180°)

$100° + \angle D = 180° \Rightarrow \angle ADC = 80°$

20. (a) 24 cm

21. (a)



In a triangle ΔAMO,

$AM = \sqrt{(10)^2 - (6)^2} = 8$

Therefore, the length of the another chord A'B' = 8 cm.

Now, $A'N = 4$

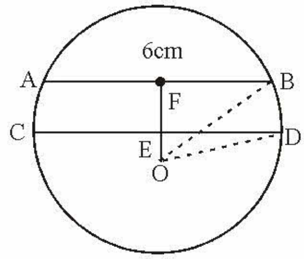
In $\Delta OA'N$,

$ON^2 = (OA')^2 - (A'N)^2$

$= 10^2 - 4^2 = 100 - 16 = 84$

$\Rightarrow ON = \sqrt{84}$

22. (a) Draw $OE \perp CD$ and $OF \perp AB$



$AB \parallel CD$ (Given)

Let 'r' be the radius of the circle

Now in rt. ΔOED,

$(OD)^2 = (OE)^2 + (ED)^2$

(using Pythagoras theorem)

$r^2 = x^2 + (6)^2 \quad \left(\therefore ED = \frac{1}{2}CD = \frac{1}{2} \times 12 = 6cm\right)$

$\Rightarrow r^2 = x^2 + 36$ ....(1)

In rt. ΔOFB, $(OB)^2 = (OF)^2 + (FB)^2$

$\Rightarrow r^2 = (x+3)^2 + (3)^2$

$\Rightarrow r^2 = x^2 + 6x + 9 + 9$

$\Rightarrow r^2 = x^2 + 6x + 18$ ....(2)

From (1) and (2), we get

$x^2 + 36 = x^2 + 6x + 18$

$\Rightarrow 36 = 6x + 18$

$\Rightarrow 36 - 18 = 6x$

$18 = 6x \Rightarrow 3 = x$

For (1),

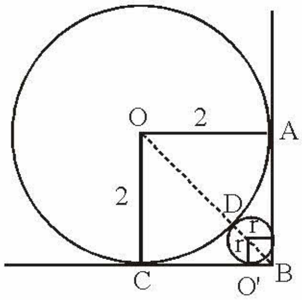
$r^2 = (3)^2 + (6)^2$

$r^2 = 9 + 36$

$\Rightarrow r^2 = 45$

$r = \sqrt{45} \Rightarrow r = 3\sqrt{5}$ cm.

23. (d)



OABC is square with side = 2

$\therefore OB = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$OB = 2\sqrt{2} = OD + r + O'B$

$= 2 + r + r\sqrt{2}$

$\Rightarrow r(\sqrt{2}+1) = 2(\sqrt{2}-1)$

$$\Rightarrow r = \frac{2(\sqrt{2}-1)}{(\sqrt{2}+1)} = \frac{2(\sqrt{2}-1)^2}{2-1} = 2(2+1-2\sqrt{2})$$

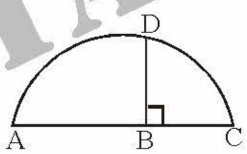
$= 6 - 4\sqrt{2}$

24. (a) $\angle EDC = \angle BAD = 45^0$(alternate angles)

$\therefore x = DEC = 180^\circ - (50^\circ + 45^\circ) = 85^\circ$.

25. (a) m $\angle$ ADC = 90^0....

(Angle subtended by the diameter on a circle is 90°)



$\therefore$ Δ ADC is a right angled triangle.

$\therefore$ $(DB)^2 = BA \times BC$ ..(DB is the perpendicular to the hypotenuse)

$= 9 \times 4 = 36$ $\therefore$ DB = 6

26. (a) Diameter of circle = x

$\therefore$ y = 4x

$\therefore$ $x = \frac{1}{4}y$.

27. (b) As F is the mid-point of AD, CF is the median of the triangle ACD to the side AD.

Hence area of the triangle FCD = area of the triangle ACF.

Similarly area of triangle BCE

= area of triangle ACE.

$\therefore$ Area of ABCD = Area of

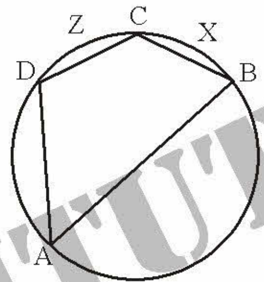
(CDF + CFA + ACE + BCE)

= 2 Area (CFA + ACE) = 2 × 13 = 26 sq. units.

28. (c) m $\angle$ DAB + 180° – 120° =60°

...(Opposite angles of a cyclic quadrilateral)

m (arc BCD) = 2m $\angle$ DAB = 120°.



$\therefore$m (arc CXB) = m (BCD) – m (arc DZC)

= 120° – 70° = 50° .

29. (c) Let the line m cut AB and CD at point P and Q respectively

$\angle$ DOQ = x (exterior angle)

Hence, Y + 2x (corresponding angle)

$\therefore$ y = x ....(1)

Also . $\angle$ DOQ = x (vertically opposite angles)

In Δ OCD, sum of the angles = 180^0

$\therefore$ y + 2y + 2x + x =180°

3x + 3y = 180°

x + y = 60 ...(2)

From (1) and (2)

x = y = 30 = 2y = 60

$\therefore$ $\angle$ ODS = 180 – 60 = 120^0

$\therefore$ θ = 180 – 3x = 180 – 3(30)

= 180 – 90 = 90°.

$\therefore$ The required ratio = 90 : 120 = 3 : 4.