CH 6 MENSURATION

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (c) According to question, circumference of circle = Area of circle

or $\pi d = \pi\left(\frac{d}{2}\right)^2$

[where d = diameter]

$\therefore$ d = 4

2. (a) In a parallelogram.

Area = Diagonal × length of perpendicular on it.

= 30 × 20 = 600 m^2\

3. (b) Required area covered in 5 revolutions

= 5 × 2πrh

$= 5 \times 2 \times \frac{22}{7} \times 0.7 \times 2 = 44\ m^2$

4. (c) In a triangle,

Area $= \frac{1}{2} \times$ length of perpendicular × base

or $615 = \frac{1}{2} \times$ length of perpendicular × 123

$\therefore$ Length of perpendicular

$= \frac{615 \times 2}{123} = 10$ m.

5. (a) In an isoscele right angled triangle,

Area = 1/23.3 × perimeter^2

= 1/23.3 × 20^2 = 17.167 m^2

6. (a) Area of rhombus = side × height

= 13 × 20

= 260 cm^2

7. (a) In a circle, circumference = 2πr

Hence, 44 = 2πr

$\therefore r = \frac{44}{2\pi}$

Now, area of circle = πr^2

$= \pi \times \frac{44}{2\pi} \times \frac{44}{2\pi} = 154\ m^2$

8. (a) Let the length and breadth of a rectangle are 9 xm and 5 xm respectively.

In a rectangle, area = length × breadth

$\therefore$ 720 = 9x × 5x

or $x^2 = 16 \Rightarrow x = 4$

Thus, length = 9 × 4 = 36 m

and breadth = 5 × 4 = 20 m

Therefore, perimeter of rectangle = 2(36 + 20) = 112 m

9. (d) Required no. of squares $= \frac{5^2}{1^2} = 25$

10. (c) Let the area of two squares be 9x and x respectively.

So, sides of both squares will be

$\sqrt{9x}$ and $\sqrt{x}$ respectively.

[since, side = $\sqrt{\text{area}}$]

Now, perimeters of both squares will be

$4 \times \sqrt{9x}$ and $4\sqrt{x}$ respectively.

[since , perimeter = 4 × side]

Thus, ratio of their perimeters $= \frac{4\sqrt{9x}}{4\sqrt{x}} = 3 : 1$

11. (a) Volume of the bucket = volume of the sand emptied

Volume of sand = $\pi (21)^2 \times 36$

Let r be the radius of the conical heap.

Then, $\frac{1}{3}\pi r^2 \times 12 = \pi(21)^2 \times 36$

or $r^2 = (21)^2 \times 9$

or $r = 21 \times 3 = 63$

12. (a) Let inner radius of the pipe be r.

Then, $440 = \frac{22}{7} \times r^2 \times 7 \times 10$

or $r^2 = \frac{440}{22 \times 10} = 2$

or $r = \sqrt{2}$ m

13. (c) Area of field = 576 km^2. Then,

each side of field = $\sqrt{576} = 24$ km

Distance covered by the horse

= Perimeter of square field

= 24 × 4 = 96 km

$\therefore$ Time taken by horse = $\frac{\text{distance}}{\text{speed}} = \frac{96}{12} = 8$ h

14. (c) Clearly, we have :

$l = 9$ and $l + 2b = 37$ or $b = 14$.

$\therefore$ Area = $(l \times b) = (9 \times 14)$ sq. ft.

= 126 sq. ft.

15. (b) We have : $2b + l = 30$

$\Rightarrow$ $l = 30 - 2b$.

Area = 100 m^2

$\Rightarrow$ $l \times b = 100$

$\Rightarrow$ $b(30 - 2b) = 100$

$\Rightarrow$ $b^2 - 15b + 50 = 0$

$\Rightarrow$ $(b - 10)(b - 5) = 0$

$\Rightarrow$ $b = 10$ or $b = 5$.

When $b = 10$, $l = 10$ and when $b = 5$, $l = 20$.

Since the garden is rectangular,

so its dimension is 20 m × 5 m.

16. (c) Area of the field

= 13.5 × 2.5 = 33.75 m^2

Area covered by the rectangular tank

= 5 × 4.5 = 22.50 m^2

Area of the field on which the earth dug out is to be spread = 33.75 – 22.50 = 11.25 m^2

Let the required height be h.

Then, 11.25 × h = 5 × 4.5 × 2.1

or h = 4.2 m

17. (b) Area of the field grazed

$= \left(\frac{22}{7} \times 14 \times 14\right)$ sq. ft.

= 616 sq. ft.

Number of days taken to graze the field

$= \frac{616}{100}$ days = 6 days (approx.)

18. (a) Volume of the water running through pipe per hour

$= \frac{20}{100} \times \frac{20}{100} \times 15000 = 600$ cubic metre

Required time

$= \frac{60 \times 6.5 \times 80}{600} = 52$ hours

19. (c) Length of wire

$= 2\pi \times R = \left(2 \times \frac{22}{7} \times 56\right)$ cm

= 352 cm.

Side of the square

$= \frac{352}{4}$ cm = 88 cm.

Area of the square = (88 × 88) cm^2 = 7744 cm^2.

20. (a) Let the edge of the third cube be x cm.

Then, $x^3 + 6^3 + 8^3 = 12^3$

$\Rightarrow$ $x^3 + 216 + 512 = 1728$

$\Rightarrow$ $x^3 = 1000$

$\Rightarrow$ $x = 10$.

Thus the edge of third cube = 10 cm.

21. (b) Area of the inner curved surface of the well dug

$= [2\pi \times 3.5 \times 22.5] = 2 \times \frac{22}{7} \times 3.5 \times 22.5$

= $44 \times 0.5 \times 22.5 = 495$ sq. m.

$\therefore$ Total cost = 495×3 = ₹ 1485.

22. (a) In a cube,

Area = 6 $(\text{side})^2$

or 150 = 6 $(\text{side})^2$

$\therefore$ side = $\sqrt{25} = 5$ m

Length of diagonal = $\sqrt{3} \times \text{side} = 5\sqrt{3}$ m

23. (c) Required length = length of the diagonal

$= \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$ m

24. (c) In a sphere, volume $= \frac{4}{3}\pi r^3$

and surface area $= 4\pi r^2$

According to question,

$\frac{4}{3}\pi r^3 \div 4\pi r^2 = 27$

or r = $27 \times 3 = 81$ cms

25. (a) Let depth of rain be h metre. Then,

volume of water

= area of rectangular field × depth of rain

or 3000 = $500 \times 300 \times h$

$\therefore h = \frac{3000}{500 \times 300}$ m

$= \frac{3000 \times 100}{500 \times 300}$ cms = 2 cms

26. (a) Area of the wet surface

= $[2(\ell b + bh + \ell h) - \ell b]$

= $2(bh + \ell h) + \ell b$

= $[2(4 \times 1.25 + 6 \times 1.25) + 6 \times 4]$ m^2

= 49 m^2.

27. (a) Internal volume

= $115 \times 75 \times 35$ = 3,01, 875 cm^3

External volume

= $(115 + 2 \times 2.5) \times (75 + 2 \times 2.5) \times (35 + 2 \times 2.5)$

= $120 \times 80 \times 40$ = 3,84,000 cm^3

$\therefore$ Volume of wood = External volume – Internal volume

= 3,84,000 – 3,01,875 = 82,125 cm^3

28. (a) Let height will be h cm.

Volume of water in roof = Volume of water in cylinder

$\Rightarrow \frac{9 \times 10000 \times 0.1}{900 \times 10} = h$

$\therefore$ h = 1 cm

29. (b) Required speed of flow of water

$= \frac{225 \times 162 \times 20}{5 \times 100} = \frac{60}{100} \times \frac{45}{100} \times h$

$\therefore$ h = 5400

30. (b) Let ℓ be the length and b be the breadth of cold storage.

L = 2B, H = 3 metres

Area of four walls

= $2[L \times H + B \times H] = 108$

$\Rightarrow$ 6BH = 108

$\Rightarrow$ B = 6

$\therefore$ L = 12, B = 6, H = 3

Volume = $12 \times 6 \times 3 = 216$ m^3

31. (b) Volume of water displace = $(3 \times 2 \times 0.01)$ m^3 = 0.06 m^3.

$\therefore$ Mass of man

= Volume of water displaced × Density of water

= (0.06×1000) kg = 60 kg.

32. (c) Let h be the required height then,

$\frac{22}{7} \times (60)^2 \times h$

$= 30 \times 60 \times \frac{22}{7} \times (1)^2 \times (600)$

$\Rightarrow$ 60 h = 30×600

$\Rightarrow$ h = 300 cm = 3 m

33. (c) Surface area of the cube

= (6×8^2) sq. ft. = 384 sq. ft.

Quantity of paint required

$= \left(\frac{384}{16}\right)$kg = 24 kg.

∴ Cost of painting

= ₹ (36.50 × 24) = ₹ 876.

34. (c) Volume of block = (6 × 9 × 12) cm^3 = 648 cm^3.

Side of largest cube

= H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of the cube = (3 × 3 × 3) = 27 cm^3.

∴ Number of cubes = $\left(\frac{648}{27}\right) = 24.$

35. (a) Total surface area of the remaining solid = Curved surface area of the cylinder + Area of the base + Curved surface area of the cone

$= 2\pi rh + \pi r^2 + \pi r \ell$

$= 2\pi \times 8 \times 15 + \pi \times (8)^2 + \pi \times 8 \times 17$

$= 240\pi + 64\pi + 136\pi$

$= 440\ \pi\ cm^2$

36. (b) L × B × 2 = 48

⇒ L × B = 24

Now, 6 – 6 × 10% = 5.4,

5 – 5 × 10% = 4.5 and

Therefore, 5.4 × 4.5 = 24.3

Clearly, 5 < L < 5.5

37. (b) Given, playground is rectangular.

Length = 36 m, Breadth = 21 m

Now, perimeter of playground

= 2(21 + 36) = 114

Now, poles are fixed along the boundary at a distance 3m.

∴ Required no. of poles = $\frac{114}{3} = 38$.

38. (a) Let the rise in water level = x m

Now, volume of pool

= 40 × 90 × x = 3600 x

When 150 men take a dip, then displacement of water = 8m^3

∴ $\frac{3600x}{150} = 8$

⇒ $\frac{900}{150}x = 2 \Rightarrow x = .33m$

⇒ x = 33.33 cm

39. (b) Dimensions of wooden box = 8m × 7m × 6m

= 800 cm × 700 cm × 600 cm

and dimensions of rectangular boxes = 8 cm × 7 cm × 6 cm

∴ No. of boxes

$= \frac{\text{Area of wooden box}}{\text{Area of rect. boxes}}$

$= \frac{800 \times 700 \times 600}{8 \times 7 \times 6} = 10,000,00$

40. (a) Let width of the field = b m

∴ length = 2 b m

Now, area of rectangular field = 2b × b = $2b^2$

Area of square shaped pond = 8 × 8 = 64

According to the question,

$64 = \frac{1}{8}(2b^2) \Rightarrow b^2 = 64 \times 4 \Rightarrow b = 16m$

∴ length of the field = 16 × 2 = 32 m

41. (a) Length of the wire = Perimeter of the circle

= 2π × 28

= 176 cm^2

Side of the square $= \frac{176}{4} = 44$ cm

42. (b) Let length, breadth and height of the room be ℓ, b and h, respectively. Then,

$\ell + b + h = 19$ ...(i)

and $\sqrt{\ell^2 + b^2 + h^2} = 11$

⇒ $\ell^2 + b^2 + h^2 = 121$ ...(ii)

Area of the surface to be painted

$= 2(\ell b + bh + h\ell)$

$(\ell + b + h)^2 = \ell^2 + b^2 + h^2 + 2(\ell b + bh + h\ell)$

$\Rightarrow \quad 2(\ell b + bh + h\ell)$

$= (19)^2 - 121 = 361 - 121 = 240$

Surface area of the room $= 240\ m^2$.

Cost of painting the required area

$= 10 \times 240 =$ ₹ 2400

43. (d) Area of the quadrilateral PQRS

= Area of ΔSPR + Area of ΔPQR

$= \frac{1}{2} \times PR \times AP + \frac{1}{2} \times PR \times PB$

$= \frac{1}{2} \times PR(AP + PB) = \frac{1}{2} \times AD \times AB$

($\because$ PR = AD and AP + PB = AB)

$= \frac{1}{2} \times$ Area of rectangle ABCD

$= \frac{1}{2} \times 16 = 8\,cm^2$

44. (a) Area of the field

$= 42 \times 35 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (21)^2 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (17.5)^2$

$= 1470 + 1386 + 962.5 = 3818.5\ m^2$

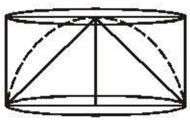
45. (b) We have,

radius of the hemisphere = raidus of the cone

= height of the cone

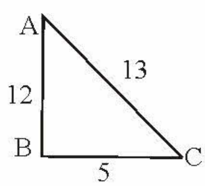
= height of the cylinder = r (say)

Then, ratio of the volumes of cylinder, hemisphere and cone



$= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3 = 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$

46. (b)



ABC forms a right angled triangle

$\therefore$ Area $= \frac{1}{2} \times 12 \times 5 = 30$

Area of rectangle $= 30 = \ell \times 10$ or $\ell = 3$ units

$\therefore$ Perimeter = 2 (10 +3) = 26

EXERCISE 2

1. (d) Perimeter of the circle = $2\pi r = 2(18 + 26)$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14$

$\therefore$ Area of the circle

$= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ cm^2$.

2. (b) Length of the carpet $= \left(\frac{\text{Total cost}}{\text{Rate / m}}\right)$

$= \left(\frac{8100}{45}\right) m = 180$ m.

Area of the room = Area of the carpet =

$\left(180 \times \frac{75}{100}\right) m^2 = 135\ m^2$.

$\therefore$ Breadth of the room

$= \left(\frac{\text{Area}}{\text{Length}}\right) = \left(\frac{135}{18}\right) m = 7.5$ m.

3. (a) In a rectangle,

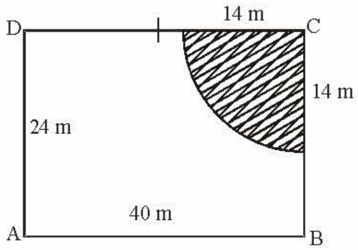
$\frac{(\text{perimeter})^2}{4} = (\text{diagonal})^2 + 2 \times \text{area}$

$\Rightarrow \quad \frac{(14)^2}{4} = 5^2 + 2 \times \text{area}$

$49 = 25 + 2 \times$ area

$\therefore$ Area $= \frac{49 - 25}{2} = \frac{24}{2} = 12\,cm^2$

4. (a)



Area of the shaded portion

$= \frac{1}{4} \times \pi (14)^2$ = 154 m^2

5. (a) Circumference of circular bed = 30 cm

Area of circular bed $= \frac{(30)^2}{4\pi}$

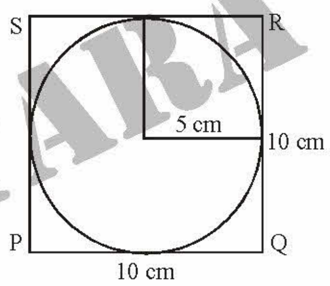
Space for each plant = 4 cm^2

∴ Required number of plants

$= \frac{(30)^2}{4\pi} \div 4 = 17.89 = 18$ (Approx)

6. (a) Area of the square = $(10)^2$ = 100 cm^2

The largest possible circle would be as shown in the figure below :



Area of the circle $= \frac{22}{7} \times (5)^2 = \frac{22 \times 25}{7}$

Required ratio $= \frac{22 \times 25}{7 \times 100} = \frac{22}{28} = \frac{11}{14}$

$= 0.785 \approx 0.8 = \frac{4}{5}$

7. (d) Side of square carpet

$= \sqrt{\text{Area}} = \sqrt{169} = 13\text{m}$

After cutting of one side,

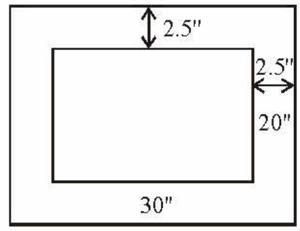
Measure of one side = 13 – 2 = 11 m

and other side = 13 m (remain same)

∴ Area of rectangular room

= 13 × 11 = 143 m^2

8. (a)



Length of frame = 30 + 2.5 × 2 = 35 inch

Breadth of frame = 20 + 2.5 × 2 = 25 inch

Now, area of picture = 30 × 20 = 600 sq. inch

Area of frame = (35 × 2.5) + (25 × 2.5) = 150

9. (a) If area of a circle decreased by x % then the radius of a circle decreases by

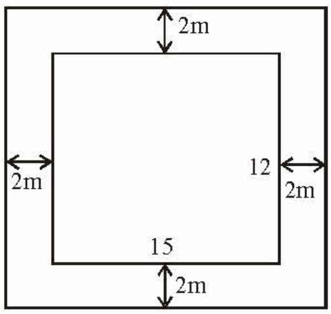
$(100 - 10\sqrt{100 - x})\%$

$= (100 - 10\sqrt{100 - 36})\%$

$= (100 - 10\sqrt{64})\%$

$= 100 - 80 = 20\%$

10. (a) Area of the outer rectangle = 19 × 16 = 304 m^2



Area of the inner rectangle = 15 × 12 = 180 m^2

Required area = (304 – 180) = 124 m^2

11. (a) Area of paper = 12 × 5 = 60 sq. inch

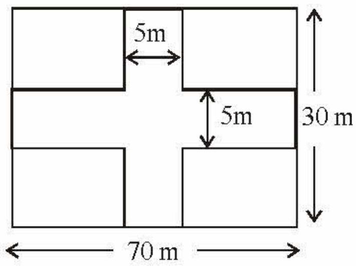
Area of typing part $= (12 - 1 \times 2) \times (5 - \frac{1}{2} \times 2)$

= (12 – 2) × (5 – 1)

= (10 × 4) sq. inch

$\therefore$ Required fraction $= \frac{40}{60} = \frac{2}{3}$

12. (c)



Total area of road

= Area of road which parallel to length + Area of road which parallel to breadth – overlapped road

= 70 × 5 + 30 × 5 – 5 × 5

= 350 + 150 – 25

= 500 – 25 = 475 m^2

$\therefore$ Cost of gravelling the road

= 475 × 4 = ₹ 1900

13. (a) Radius of a circular grass lawn (without path) = 35 m

$\therefore$ Area $= \pi r^2 = \pi (35)^2$

Radius of a circular grass lawn (with path)

= 35 + 7 = 42 m

$\therefore$ Area $= \pi r^2 = \pi(42)^2$

$\therefore$ Area of path $= \pi(42)^2 - \pi(35)^2$

$= \pi(42^2 - 35^2)$

$= \pi(42 + 35)(42 - 35)$

$= \pi \times 77 \times 7$

$= \frac{22}{7} \times 77 \times 7 = 1694\,m^2$

14. (b) Radius of the wheel of bus = 70 cm. Then,

circumference of wheel

$= 2\pi r = 140\pi = 440$ cm

Distance covered by bus in 1 minute

$= \frac{66}{60} \times 1000 \times 100$ cms

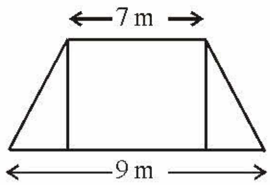
Distance covered by one revolution of wheel

= circumference of wheel

= 440 cm

$\therefore$ Revolutions per minute $= \frac{6600000}{60 \times 440} = 250$

15. (a)



Let the length of canal = h m. Then,

area of canal $= \frac{1}{2} \times h(9+7)$

or $1280 = \frac{1}{2}h(16)$

$\therefore h = \frac{1280 \times 2}{16} = 160$ m

16. (a) When folded along breadth, we have :

$2\left(\frac{l}{2} + b\right) = 34$ or $l + 2b = 34$ ...(i)

When folded along length, we have :

$2\left(l + \frac{b}{2}\right) = 38$ or $2l + b = 38$ ....(ii)

Solving (i) and (ii), we get :

$l = 14$ and $b = 10$.

$\therefore$ Area of the paper = (14 × 10) cm^2 = 140 cm^2.

17. (a) Area left after laying black tiles

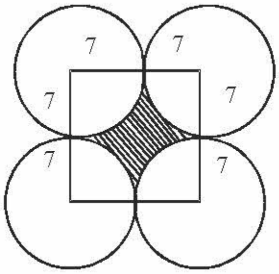
= [(20 – 4) × (10 – 4)] sq. ft. = 96 sq. ft.

Area under white tiles $= \left(\frac{1}{3} \times 96\right)$ sq. ft = 32 sq. ft.

Area under blue tiles = (96 – 32) sq. ft = 64 sq. ft.

Number of blue tiles $= \frac{64}{(2 \times 2)} = 16.$

18. (b)



The shaded area gives the required region.

Area of the shaded region

= Area of the square – area of four quadrants of the circles

$$= (14)^2 - 4 \times \frac{1}{4} \pi (7)^2$$

$$= 196 - \frac{22}{7} \times 49 = 196 - 154 = 42\,cm^2$$

19. (b) Perimeter = Distance covered in 8 min.

$$= \left(\frac{12000}{60} \times 8\right) m = 1600 \text{ m.}$$

Let length = 3x metres and breadth = 2x metres.

Then, 2 (3x + 2x) = 1600 or x = 160.

∴ Length = 480 m and Breadth = 320 m.

∴ Area = $(480 \times 320)\ m^2 = 153600\ m^2$.

20. (c) Let h = 2x metres and (*l* + b) = 5x metres.

Length of the paper

$$= \frac{\text{Total cost}}{\text{Rate per m}} = \frac{260}{2} \text{ m} = 130 \text{ m.}$$

Area of the paper

$$= \left(130 \times \frac{50}{100}\right) m^2 = 65\,m^2.$$

Total area of 4 walls

= $(65 + 15)\ m^2 = 80\ m^2$.

∴ $2(l + b) \times h = 80$

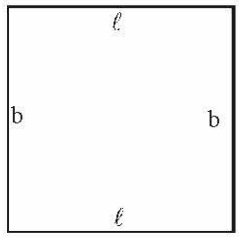
⇒ $2 \times 5x \times 2x = 80$

⇒ $x^2 = 4 \Rightarrow x = 2$.

∴ Height of the room = 4 m.

21. (b) $\ell \times b = 100\ m^2$

$$\Rightarrow \ell = \frac{100}{b}$$



Therefore, $\frac{100}{b} + b + b = 30$

or $b^2 - 15b + 50 = 0$

$b = 10, 5$

If we take b = 10, then garden becomes a square

Therefore, b = 5 m.

22. (a) Let the length of the room be ℓ m

Then its, breadth = $\ell/2$

Therefore, $\ell \times \frac{\ell}{2} = \frac{5000}{25}$

or $\ell^2 = 400$

or $\ell = 20$ m

Also, $2\ell h + 2 \times \frac{\ell}{2} \times h = \frac{64800}{240}$

⇒ $3\ell h = 270$

or $h = \frac{270}{3 \times 20} = \frac{270}{60} = 4.5\,m$

23. (c) Let each wheel make x revolutions per sec. Then,

$$\left[\left(2\pi \times \frac{7}{2} \times x\right) + (2\pi \times 7 \times x)\right] \times 10 = 1980$$

$$\Rightarrow \left(\frac{22}{7} \times 7 \times x\right) + \left(2 \times \frac{22}{7} \times 7 \times x\right) = 198$$

⇒ 66x = 198 ⇒ x = 3.

Distance moved by smaller wheel in 3 revolutions

$$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 3\right) \text{ cm} = 66 \text{ cm.}$$

$\therefore$ Speed of smaller wheel

$= \frac{66}{3}$ cm/s = 22 cm/s.

24. (d) Let the length, breadth and height of the cuboid be x, 2x and 3x, respectively.

Therefore, volume = x × 2x × 3x = $6x^3$

New length, breadth and height = 2x, 6x and 9x, respectively.

New volume = $108x^3$

Thus, increase in volume = $(108 - 6)x^3 = 102\ x^3$

$$\frac{\text{Increase in volume}}{\text{Original volume}} = \frac{102x^3}{6x^3} = 17$$

25. (d) Let the length of the wire be h cm.

and radius of sphere and wire are R and r respectively.

Then, volume of sphere = volume of wire (cylinder)

or $\frac{4}{3}\pi R^3 = \pi r^2 h$

or $\frac{4}{3}R^3 = r^2 h$

or $\frac{4}{3}(3)^3 = (0.1)^2 h$

$\therefore$ $h = \frac{4\times(3)^3}{3\times(0.1)^2} = \frac{108}{0.03} = 3600\text{cm} = 36$ m

26. (c) Let the depth of the drainlet be h metres.

Volume of the earth dyg from the drainlet 10 m wide

= h[260 × 200 – 240 × 180]

= 8800 h cu. m.

Now this is spread over the plot raising its height by 25 cm,

i.e., $\frac{1}{4}$m.

$\therefore$ 8800 h = 240 × 180 × $\frac{1}{4}$

$\Rightarrow$ $h = \frac{60\times180}{8800} = \frac{27}{22}$

$\therefore$ h = 1.227 m.

27. (b) Volume required in the tank = (200 × 150 × 2) m^3

= 60000 m^3.

Length of water column flown in 1 min.

$= \left(\frac{20\times1000}{60}\right)m = \frac{1000}{3}m.$

Volume flown per minute =

$\left(1.5\times1.25\times\frac{1000}{3}\right)m^3$

= 625 m^3.

$\therefore$ Required time = $\left(\frac{60000}{625}\right)$min. = 96min.

28. (c) Volume of cylinder = $(\pi \times 6 \times 6 \times 28)cm^3 = (36 \times 28)\pi\ cm^3$.

Volume of each bullet

$= \left(\frac{4}{3}\pi\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\right)cm^3$

$= \frac{9\pi}{16}cm^3.$

Number of bullets

$= \frac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$

$= \left[(36\times28)\pi\times\frac{16}{9\pi}\right] = 1792.$

29. (a) Volume of water in the reservoir

= area of empty pipe × Empty rate × time to empty

or 54 × 44 × 10

$= \pi\times\left(3\times\frac{1}{100}\right)^2\times20\times\text{empty time}$

or Empty time = $\frac{54\times44\times10\times100\times100\times7}{22\times20\times9}$ sec.

$= \dfrac{54 \times 44 \times 10 \times 100 \times 100 \times 7}{22 \times 20 \times 9 \times 3600}$ hrs

= 116.67 hours.

30. (a) Let radius of the 3rd spherical ball be R,

$\therefore \quad \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi R^3$

$\Rightarrow \quad R^3 = \left[\left(\frac{3}{2}\right)^3 - \left(\frac{3}{4}\right)^3\right] - 1^3$

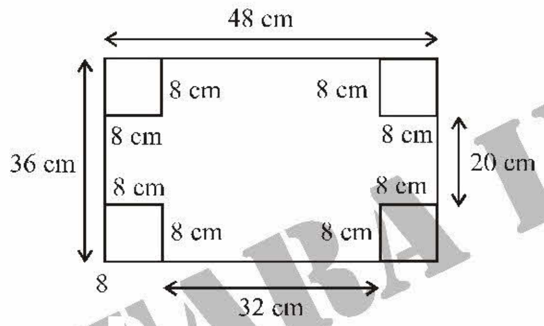
$= \frac{27}{8} - \frac{27}{64} - 1 = \frac{125}{64} = \left(\frac{5}{4}\right)^3 \Rightarrow R = \frac{5}{4} = 1.25$

$\therefore$ Diameter of the third spherical ball

= 1.25 × 2 = 2.5 cm.

31. (c) Volume of the box made of the remaining sheet

= 32 × 20 × 8 = 5120 cm^3



32. (c) Let 'A' be the side of bigger cube and 'a' be the side of smaller cube

Surface area of bigger cube = 6 A^2

or 384 = 6A^2

$\therefore$ A = 8 cm.

Surface area of smaller cube = 6 a^2

96 = 6a^2

$\therefore$ a = 4 mm = 0.4 cm

So, Number of small cube

$= \dfrac{\text{Volume of bigger cube}}{\text{Volume of smaller cube}}$

$= \dfrac{(8)^3}{(0.4)^3} = \dfrac{512}{0.064} = 8{,}000$

33. (c) Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

$= \left(\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 50\right) cm^3$

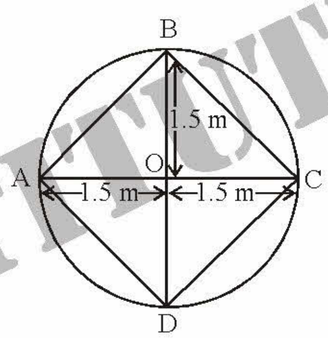
$= \left(\frac{22 \times 4 \times 12 \times 50}{7}\right) cm^3$.

Let the height of the liquid in the vessel be h.

Then, $\frac{22}{7} \times 10 \times 10 \times h = \frac{22 \times 4 \times 12 \times 50}{7}$

or $\quad h = \left(\frac{4 \times 12 \times 50}{10 \times 10}\right) = 24$ cm.

34. (c)



From ΔAOB,

$AB = \sqrt{1.5^2 + 1.5^2} = \sqrt{2.25 + 2.25} = \sqrt{4.50}$

$\therefore$ Area of the square base of the trunk of the tree

$= \sqrt{4.50} \times \sqrt{4.50} = 4.50\ m^2$

$\therefore$ Volume of the timber = Area of base × height

= 4.50 × 10 = 45 m^3

35. (d) Volume of the tank = 246.4 litres = 246400 cm^3.

Let the radius of the base be r cm. Then,

$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400$

$\Rightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Rightarrow r = 14.$

$\therefore$ Diameter of the base = 2r = 28 cm = .28 m

36. (c) Let the radius of the base are 5k and 12k respectively

$\therefore \dfrac{\text{Total surface area of the cylinder}}{\text{Total surface area of the cone}}$

$= \dfrac{2\pi r \times h + 2\pi r^2}{\pi r\sqrt{r^2+h^2} + \pi r}$

$= \dfrac{2h+2r}{\sqrt{r^2+h^2}+r} + \dfrac{24k+10k}{\sqrt{25k^2+144k^2}+5k}$

$= \dfrac{34k}{13k+5k} = \dfrac{34k}{18k} = \dfrac{17}{9}$

37. (a) Number of discharge pipe

$= \dfrac{\text{Volume of water supply pipe}}{\text{Volume of water in each discharge pipe}}$

$= \dfrac{\pi\times(3)^2\times 1}{\pi\times\left(\dfrac{3}{2}\right)^2\times 1} = 4$

[Since the velocity of water is same]

38. (c) Volume of one coin

$= \left(\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}\times 200\right)\text{cm}^3 = 7700\ \text{cm}^3.$

Volume of water flown in 10 min. = $(7700 \times 60 \times 10)\ \text{cm}^3$

$= \left(\dfrac{7700\times 60\times 10}{1000}\right)$ litres

= 4620 litres.

39. (d) $4\pi(r+2)^2 - 4\pi r^2 = 352$

$\Rightarrow (r+2)^2 - r^2 = \left(352\times\dfrac{7}{22}\times\dfrac{1}{4}\right) = 28.$

$\Rightarrow (r+2+r)(r+2-r) = 28$

$\Rightarrow 2r+2 = \dfrac{28}{2} \Rightarrow 2r+2 = 14 \Rightarrow r = 6$ cm

40. (a) Let length, breadth and height of the room be ℓ, b and h, respectively.

Then, area of four walls of the room

$= 2(\ell + b)h = \dfrac{340.20}{1.35} = 252\text{m}^2$

$\Rightarrow (\ell + b)h = 126$ ...(i)

And $\ell \times b = \dfrac{91.8}{0.85} = 108$

$12 \times b = 108$ ($\because \ell = 12$ m)

$\Rightarrow b = 9$ m

Using (i), we get, $h = \dfrac{126}{21} = 6$ m

41. (b) Volume of material in the sphere

$= \left[\dfrac{4}{3}\pi\times\left\{(4)^3-(2)^3\right\}\right]\text{cm}^3 = \left(\dfrac{4}{3}\pi\times 56\right)\text{cm}^3.$

Let the height of the cone be h cm.

Then, $\dfrac{1}{3}\pi\times 4\times 4\times h = \left(\dfrac{4}{3}\pi\times 56\right)$

$\Rightarrow h = \left(\dfrac{4\times 56}{4\times 4}\right) = 14$ cm.

42. (d) Volume of sphere = $\left(\dfrac{4}{3}\pi\times 9\times 9\times 9\right)\text{cm}^3.$

Volume of cone = $\left(\dfrac{1}{3}\pi\times 9\times 9\times 9\right)\text{cm}^3.$

Volume of wood wasted

$= \left[\left(\dfrac{4}{3}\pi\times 9\times 9\times 9\right) - \left(\dfrac{1}{3}\pi\times 9\times 9\times 9\right)\right]\text{cm}^3.$

$= (\pi\times 9\times 9\times 9)\ \text{cm}^3$

$\therefore$ Required percentage =

$\left(\dfrac{\pi\times 9\times 9\times 9}{\dfrac{4}{3}\times\pi\times 9\times 9\times 9}\times 100\right)\%$

$= \left(\dfrac{3}{4}\times 100\right)\% = 75\%.$

43. (c) Let the height of the vessel be x.

Then, radius of the bowl

= radius of the vessel = x/2.

Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{12}\pi x^3$.

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3$.

Since $V_2 > V_1$, so the vessel can contain 100% of the beverage filled in the bowl.

44. (b) Radius of the inner track = 100 m

and time = 1 min 30 sec ≡ 90 sec.

Also, Radius of the outer track = 102 m

and time = 1 min 32 sec ≡ 92 sec.

Now, speed of A who runs on the inner track

$= \frac{2\pi(100)}{90} = \frac{20\pi}{9} = 6.98$

And speed of B who runs on the outer track

$= \frac{2\pi(102)}{90} = \frac{51\pi}{23} = 6.96$

Since, speed of A > speed of B

∴ A runs faster than B.

45. (b) Curved surface area of cylinder = 2πrh

∴ Surface area of 50 cylindrical pillars

= 50 × 2πrh

Now, Diameter of each cylindrical pillar = 50 cm

∴ Radius = $\frac{50}{2}$ = 25 cm ≈ .25 m

Also, height = 4m

∴ Surface area= 50 × 2 × 3.14 ×. 25 × 4

= 314 × 1 sq m.

= 314 sq. m.

Now, labour charges at the rate of 50 paise

per sq. m = 314 ×.5 = 157.0

≡ ₹ 157

46. (c) Given, length of garden = 24 m and

breadth of garden = 14 m

∴ Area of the garden = 24 × 14 m^2 = 336 m^2.

Since, there is 1 m wide path outside the garden

∴ Area of Garden (including path)

= (24 + 2) × (14 + 2)

= 26 × 16 m^2 = 416 m^2.

Now, Area of Path

= Area of garden(inculding path)

– Area of Garden

= 416 – 336 = 80 m^2.

Now , Area of Marbles = 20 × 20 = 400 cm^2

∴ Marbles required = $\frac{\text{Area of Path}}{\text{Area of Marbles}}$

$= \frac{80,0000}{400} = 2000$

47. (c) Volume of rain that is to be collected

in a pool = $2 \times 1 \times 10^{10} \times \frac{1}{2}$

= 10^{10} cm = 10^4 meter

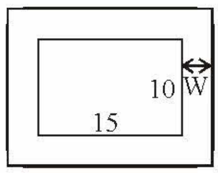
Volume of Pool = L × B × h

10^4 = 100 × 10 × h

h = $\frac{10^4}{100 \times 10} = 10\text{m}$.

EXERCISE 3

1. (c)



Let the width of the path = W m

then, length of plot with path = (15 + 2W) m

and breadth of plot with path = (10 + 2 W) m

Therefore, Area of rectangular plot (wihout path)

= 15 × 10 = 150 m^2

and Area of rectangular plot (with path)

= 150 + 54 = 204 m^2

Hence, (15 + 2W) × (10 + 2W) = 204

⇒ $4W^2 + 50W - 54 = 0$

$\Rightarrow 2W^2 + 25\,W - 27 = 0$

$\Rightarrow (W - 2)\,(W + 27) = 0$

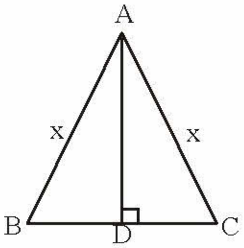
Thus W = 2 or –27

$\therefore$ with of the path = 2 m

2. (b) Let ABC be the isosceles triangle and AD be the altitude.

Let AB = AC = x.

Then, BC = (32 – 2x).



Since, in an isosceles triangle, the altitude bisects the base. So, BD = DC = (16 – x).

In ΔADC, $AC^2 = AD^2 + DC^2$

$\Rightarrow x^2 = (8)^2 + (16 - x)^2$

$\Rightarrow 32x = 320 \Rightarrow x = 10.$

$\therefore$ BC = (32 – 2x) = (32 – 20) cm = 12 cm.

Hence, required area

$$= \left(\frac{1}{2}\times BC\times AD\right)$$

$$= \left(\frac{1}{2}\times 12\times 10\right) cm^2 = 60\ cm^2.$$

3. (c) Let the length and breadth of the original rectangular field be x m and y m respectively.

Area of the original field = $x \times y = 144\ m^2$

$$\therefore\ x = \frac{144}{y} \qquad \ldots (i)$$

If the length had been 6 m more, then area will be

$(x + 6)\,y = 144 + 54$

$\Rightarrow (x + 6)\,y = 198 \qquad \ldots (ii)$

Putting the value of x from eq (i) in eq (ii), we get

$$\left(\frac{144}{y} + 6\right)y = 198$$

$\Rightarrow 144 + 6y = 198$

$\Rightarrow 6y = 54 \Rightarrow y = 9$ m

Putting the value of y in eq (i) we get x = 16 m

4. (b) Volume of the cylinder container

$= \pi \times 6^2 \times 15$ cu. cm $\qquad \ldots(1)$

Let the radius of the base of the cone be r cm,

then, height of the cone = 4r cm

$\therefore$ Volume of the 10 cylindrical cones of ice-cream with hemispherical tops

$$= 10\times\left[\frac{1}{3}\times\pi\times r^2\times 4r\right] + 10\times\frac{2}{3}\pi r^3$$

$$= \frac{40}{3}\pi r^3 + \frac{20}{3}\pi r^3$$

$= 20\,\pi r^3$ cu. cm $\quad \ldots(2)$

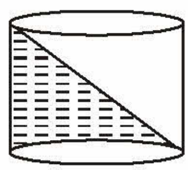
5. (d) Let the original volume of cylinder be V .

When it is filled $\frac{4}{5}$, then it's volume = $\frac{4}{5}V$

When cylinder is filled, the level of water coincides with opposite sides of bottom and top edges then

Volume become = $\frac{1}{2}V$

Since, in this process 30 cc of the water is spilled, therefore



$$\frac{4}{5}V - 30 = \frac{1}{2}V$$

$$\Rightarrow \frac{4}{5}V - \frac{1}{2}V = 30$$

$\Rightarrow V\,(3/10) = 30$

$\Rightarrow V = 100$ cc

Since the whole ice-cream in the cylindrical container is distributed among 10 children in cones with hemispherical tops,

$\therefore$ (1) and (2), gives

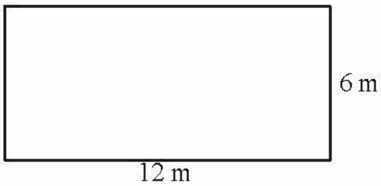
$\Rightarrow \pi \times 6^2 \times 15 = 20\pi r^3$

$\Rightarrow r^3 = \frac{36 \times 15}{20} = 27$

$\Rightarrow r = 3$ cm

6. (c) Given, Length = 12 m and Breadth = 6 m

$\therefore$ Area of rectangular plate = 12 × 6 = 72 m^2



Since, two apertures of 3 m diameter each have been made from this plate.

$\therefore$ Area of these two apertures = $\pi(1)^2 + \pi(1)^2$

$= \pi + \pi = 2\pi$

Area of 1 aperture of 1m diameter

$= \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

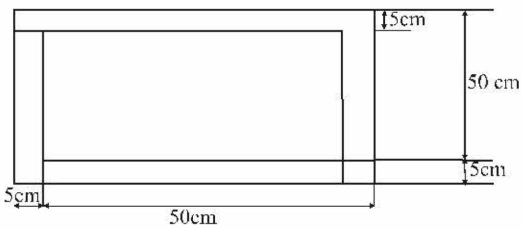
$\therefore$ Total area of aperture

$= 2\pi + \frac{\pi}{4} = \frac{9\pi}{4} = \frac{9}{4} \times \frac{22}{7} = \frac{99}{14}$

$\therefore$ Area of the remaining portion of the plate

$= 72 - \frac{99}{14}$ sq. m $= \frac{909}{14}$ sq. m ≈ 64.5 sq.m

7. (b)



Side of the inner square = 55 – 10 = 45

$\therefore$ Area of inner square = 45 × 45 = 2025 sq. m.

8. (c) Let the kerosene level of cylindrical jar be h.

Now, Volume of conical vessel

$= \frac{1}{3}\pi r^2 h$

Since, radius (r)

= 2 cm and height(h) = 3cm of conical vessel.

$\therefore$ Volume = $\frac{1}{3}\pi \times 4 \times 3 = 4\pi$

Now, Volume of cylinderical jar = $\pi r^2 h$

$= \pi (2)^2 h = 4\pi h$

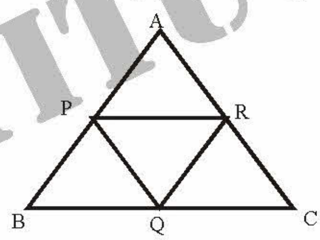
Now, Volume of conical vessel = Volume of cylindrical Jar

$\Rightarrow 4\pi = 4\pi h$

h = 1cm

Hence, kerosene level in Jar is 1 cm.

9. (c) Consider for an equilateral triangle. Hence ΔABC consists of 4 such triangles with end points on mid pts AB, BC and CA



$\Rightarrow \frac{1}{4}$ ar (ΔABC) = ar (ΔPQR)

$\Rightarrow$ ar (ΔPQR) = 5 sq. units

10. (b) Let the common base be x m.

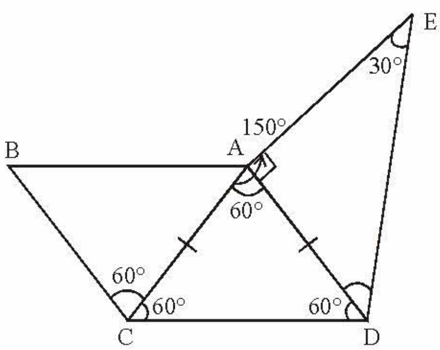
Now, area of the triangle

= area of the parallelogram

$\frac{1}{2} \times x \times$ Altitude of the triangle = x × 100

Altitude of the triangle = 200 m

11. (c)



$\angle A = \angle C = 60$ (alternative angles)

$\angle C = \angle D = 60°$ (since AC = AD and $\angle A = 60°$)

Δ ACD is equilateral

so its area $= \frac{x^2\sqrt{3}}{4}$ (where x is side)

Area of parallelogram ABCD

$= 2\times\frac{x^2\sqrt{3}}{4} = \frac{x^2\sqrt{3}}{2}$

Area of $\Delta ADE = \frac{1}{2}\times AD\times AE$

$= \frac{1}{2}\times x\times x\tan 60° = \frac{x^2\sqrt{3}}{2}$

therefore we see,

Area of parallelogram ABCD = Area of ΔADE

12. (c) $\frac{\text{Ratio of uncut portion}}{\text{Ratio of cut portion}} = \frac{(\pi\times 20\times 20)-(100\pi)}{(4\times\pi\times 5\times 5)}$

$= \frac{300\pi}{100\pi} = \frac{3}{1}$

13. (a) $AD = 24, BC = 12$

In ΔBCE & ΔADE

since $\angle CBA = \angle CDA$ (Angles by same arc)

$\angle BCE = \angle DAE$ (Angles by same arc)

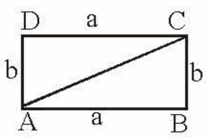
$\angle BEC = \angle DEA$ (Opp. angles)

$\therefore \angle BCE$ & $\angle DAE$ are similar Δs

with sides in the ratio 1 : 2

Ratio of area = 1:4 (i.e square of sides)

14. (c)



$AC + AB = 5AD$ or $AC + a = 5b$ ...(1)

$AC - AD = 8$ or $AC = b + 8$ ...(2)

Using (1) and (2) ,

$a + b + 8 = 5b$ or $a + 8 = 4b$ ...(3)

Using Pythagorous theorem,

$a^2 + b^2 = (b+8)^2 = b^2 + 64 + 16b$

or $a^2 = 16b + 64 = (4b-8)^2 = 16b^2 + 64 - 64b$

[From (3)]

$\Rightarrow 16b^2 - 80b = 0$ or $b = 0$ or 5

Putting b = 5 in (3),

a = 4b - 8 = 20 - 8 = 12

Area of rectangle = $12\times 5 = 60$

15. (b) In the figure $\angle ACB$ is 90°

(angle subtended by diameter= 90°)

$AC = 5, AB = 13$

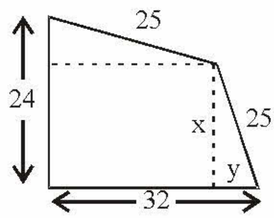
Using pythagoras theorem,

$AB^2 = AC^2 + CB^2$

$\Rightarrow CB = \sqrt{13^2 - 5^2} = 12$

Area of $\Delta ABC = \frac{1}{2}\times 5\times 12 = 30$

16. (d)



$(32-y)^2 + (24-x)^2 = 625$ ....(1)

$x^2 + y^2 = 625$ ....(2)

$\Rightarrow (24)^2 + (32)^2 - 64y - 48x = 0$

(From (1) & (2))

$\Rightarrow 64y + 48x = 576 + 1024$

$\Rightarrow 4y + 3x = 36 + 64 = 100$

or $y = \left(\frac{100-3x}{4}\right)$

$\therefore\ x^2+\left(\frac{100-3x}{16}\right)^2=625$

(From (2))

$\Rightarrow -600x+16x^2+10000+9x^2=625\times16$

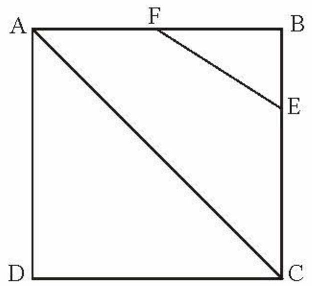
$\Rightarrow 25x^2-600x+10000-625\times16=0$

$\Rightarrow x=24$ and $y=7$

$\therefore$ Area $=(24\times25)+\frac{1}{2}\times24\times7=684$

17. (b) Let the side of the square be x, then

$BE=\frac{x}{3}$ and $BF=\frac{x}{2}$



Area of $\Delta FEB=\frac{1}{2}\times\frac{x}{3}\times\frac{x}{2}=\frac{x^2}{12}$

Now, $\frac{x^2}{12}=108$

$\Rightarrow\ x^2=108\times12=1296$

In ΔADC, we have

$AC^2=AD^2+DC^2$

$=x^2+x^2=2x^2$

$=2\times1296=2592$

or $AC=\sqrt{2592}=36\sqrt{2}$

18. (b) Area of the semi-circle $=\frac{\pi}{2}=1.571\ m^2$

Area of $\Delta ABE=\frac{1}{2}\times AB\times BE$

$=\frac{1}{2}\times3\times2=3\ m^2$

Area of rectangle BCDE

$=10\times2=20\ m^2$

Total covered area

$=1.571+3+20=24.571\ m^2$

Prize money won $=24.571\times100\approx$ ₹ 2457

19. (d) $(\text{side})^2=$

$\left(\frac{1}{2}\text{x one diagonal}\right)^2+\left(\frac{1}{2}\text{x other diagonal}\right)^2$

$13^2=\left(\frac{1}{2}\text{x one diagonal}\right)^2+\left(\frac{1}{2}\text{x } 24\right)^2$

$169-144=\left(\frac{1}{2}\text{x diagonal}\right)^2$

$25=\left(\frac{1}{2}\text{x diagonal}\right)^2$

$5=\frac{1}{2}\times\text{diagonal}$

$\therefore$ diagonal = 10

$\therefore$ Area $=\frac{1}{2}$ x 10 x 24 = 120 sq. cm.

20. (d) Let the radius of the semi- circle be R and that of the circle be r, then from the given data, it is not possible to express r in terms of R. Thus option (d) is the correct alternative.