CH 7 PERMUTATION AND COMBINATION

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (e)

O, A, E	S	F	T	W	R

When the vowels are always together, then treat all the vowels as a single letter and then all the letters can be arranged in 6! ways and also all three vowels can be arranged in 3! ways. Hence, required no. of arrangements

= 6! × 3! = 4320.

2. (a) Reqd no. of ways = ${}^7C_4 \times {}^8C_4$

$$= \frac{7\times6\times5\times4}{1\times2\times3\times4}\times\frac{8\times7\times6\times5}{1\times2\times3\times4}$$

$$= 35\times70 = 2450$$

3. (b) Treat B and T as a single letter. Then the remaining letters (5 + 1 = 6) can be arranged in 6! ways. Since, O is repeated twice, we have to divide by 2 and the B and T letters can be arranged in 2! ways.

Total no. of ways

$$= \frac{6!\times2!}{2} = 720$$

4. (a) Reqd. number of ways

$$\frac{6!}{2!\times2!} = \frac{6\times5\times4\times3}{1\times2} = 180$$

5. (b) $27^2 < 765 < 28^2$

∴ required no. of chairs to be excluded = 765 − 729 = 36

6. (a) Reqd. number = 4! × 2! = 24× 2 = 48

7. (e) The word SIGNATURE consists of nine letters comprising four vowels (A, E, I and U) and five consonants (G, N, R, T and S). When the four vowels are considered as one letter, we have six letters which can be arranged in ${}^6P_6$ ways ie 6! ways. Note that the four vowels can be arranged in 4! ways.

Hence required number of words

= 6! × 4!

= 720 × 24 = 17280

8. (c) Here, 5 men out of 8 men and 6 women out of 10 women can be chosen in ${}^8C_5 \times {}^{10}C_6$ ways, i.e., 11760 ways.

9. (b) We have $\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$

Or ${}^{56}P_{r+6} = 30800\,({}^{54}P_{r+3})$

Or $\frac{(56)!}{(56-r-6)!} = 30800\times\frac{(54)!}{(54-r-3)!}$

Or $\frac{56\times55\times(54)!}{(50-r)!} = 30800\times\frac{(54)!}{(51-r)!}$

Or $56\times55\times(51-r) = 30800$

Or $51-r = \frac{30800}{56\times55} = 10$

$\therefore r = 51-10 = 41$

10. (a) ${}^9P_5 + 5\,.\,{}^9P_4 = {}^{10}P_r$

Or $\frac{9!}{(9-5)!} + 5.\frac{9!}{(9-4)!} = \frac{(10)!}{(10-r)!}$

Or $\frac{9!}{4!} + 5.\frac{9!}{5!} = \frac{(10)!}{(10-r)!}$

Or $\frac{9!}{4!} + .\frac{9!}{4!} = \frac{(10)!}{(10-r)!}$

Or $2\times\frac{9!}{4!}=\frac{(10)!}{(10-r)!}$

Or $\frac{(10)!}{5!}=\frac{(10)!}{(10-r)!}$

$\Rightarrow 10-r=5$ or $r=5$

11. (b) The inequality is ${}^{n+1}C_3-{}^{n+1}C_2\le 100$

We must have $n+1\ge 3$ and $n+1\ge 2$

$\Rightarrow n\ge 2$ and $n\ge 1$

$\Rightarrow n\ge 2$ and also

$\frac{(n+1)n(n-1)}{6}-\frac{(n+1)n}{2}\le 100$

$\Rightarrow (n+1)n(n-4)\le 600$

By trial the values of n satisfying this are 2, 3, 4, 5, 6, 7, 8, 9 which are eight in number.

12. (b) ${}^{m}C_3+{}^{m}C_4>{}^{m+1}C_3$

$\Rightarrow {}^{m+1}C_4>{}^{m+1}C_3$

$[\because {}^{n}C_r+{}^{n}C_{r+1}={}^{n+1}C_{r+1}]$

$\Rightarrow \frac{(m+1)!}{(m-3)!\,4!}>\frac{(m+1)!}{(m-2)!\,3!}\Rightarrow m-2>4$

$\Rightarrow m>6\Rightarrow$ The least value of m is 7.

13. (b) ${}^{39}C_{3r-1}-{}^{39}C_{r^2}={}^{39}C_{r^2-1}-{}^{39}C_{3r}$

$\Rightarrow {}^{39}C_{3r-1}+{}^{39}C_{3r}={}^{39}C_{r^2-1}+{}^{39}C_{r^2}$

$\Rightarrow {}^{40}C_{3r}={}^{40}C_{r^2}$

$\Rightarrow r^2=3r$ or $r^2=40-3r$

$\Rightarrow r=0,3$ or $-8,5$

3 and 5 are the values as the given equation is not defined by r = 0 and r = –8. Hence, the number of values of r is 2.

14. (b) The thousandth place can be filled up in 9 ways with any one of the digits 1, 2, 3, ..., 9. After that the other three places can be filled up in ${}^{9}P_3$ ways, with any one of the remaining 9 digits including zero. Hence, the number of four digit numbers with distinct digits = $9\times{}^{9}P_3$.

15. (b) We have, ${}^{n}P_r={}^{n}P_{r+1}$

$\Rightarrow \frac{n!}{(n-r)!}=\frac{n!}{(n-r-1)!}\Rightarrow\frac{1}{(n-r)}=1$

or $n-r=1$ ...(1)

Also, ${}^{n}C_r={}^{n}C_{r-1}$

$\Rightarrow r+r-1=n$

$\Rightarrow 2r-n=1$ ...(2)

Solving (1) and (2), we get r = 2 and n = 3

16. (c) ${}^{n}P_r=720\,{}^{n}C_r$

or $\frac{n!}{(n-r)!}=\frac{720(n!)}{(n-r)!r!}$

$\Rightarrow r!=720=1\times2\times3\times4\times5\times6!$

or r = 6

17. (b) Since each bulb has two choices, either switched on or off, therefore required number = $2^{10}-1$ = 1023.

18. (b) Here we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. this can be done in $\frac{52!}{1!\,51!}$ ways.

Now every group of 51 cards can be divided into 3 groups of 17 each in $\frac{51!}{(17!)^3\,3!}$.

Hence the required number of ways

$=\frac{52!}{1!\,51!}\cdot\frac{51!}{(17!)^3\,3!}=\frac{52!}{(17!)^3\,3!}$

19. (b) When 0 is the repeated digit like

100, 200, ..., 9 in number

When 0 occurs only once like

110, 220, ..., 9 in number

When 0 does not occur like

112, 211, ..., $2\times(8\times9)=144$ in number.

Hence, total = 9 + 9 + 144 = 162.

20. (b) Suppose $x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7$ represents a seven digit number. Then x_1 takes the value 1, 2, 3, ..., 9 and x_2, x_3, ..., x_7 all take values 0, 1, 2, 3, ..., 9.

If we keep x_1, x_2, ..., x_6 fixed, then the sum $x_1 + x_2 + ... + x_6$ is either even or odd. Since x_7 takes 10 values 0, 1, 2, ..., 9, five of the numbers so formed will have sum of digits even and 5 have sum odd.

Hence the required number of numbers

= 9 . 10 . 10 . 10 . 10 . 10 . 5 = 4500000.

21. (a) Number of single digit numbers = 5

Number of two digits numbers = 4 × 5

[∵ 0 cannot occur at first place and repetition is allowed]

Number of three digits numbers

= 4 × 5 × 5 = 4×5^2

....

....

Number of 20 digits numbers = 4×5^{19}

∴ Total number of numbers

$= 5 + 4.5 + 4.5^2 + 4.5^3 ....4.5^{19}$

$= 5 + 4.\dfrac{5(5^{19}-1)}{5-1} = 5 + 5^{20} - 5 = 5^{20}$

22. (d) The first and the last (terminal) digits are even and there are three even digits. This arrangement can be done in $^3P_2$ ways. For any one of these arrangements, two even digits are used; and the remaining digits are 5 (4 odd and 1 even) and the four digits in the six digits (leaving out the terminal digits) may be arranged using these 5 digits in $^5P_4$ ways. The required number of numbers is $^3P_2 \times {}^5P_4 = 6 \times 120 = 720$.

23. (d) Required number of possible outcomes

= Total number of possible outcomes –

Number of possible outcomes in which 5 does not appear on any dice. (hence 5 possibilities in each throw)

$= 6^3 - 5^3 = 216 - 125 = 91$

24. (a) Ten candidates can be ranked in 10! ways. In half of these ways A_1 is above A_2 and in another half A_2 is above A_1. So, required number of ways is $\dfrac{10!}{2}$.

25. (b) Let the two boxes be B_1 and B_2. There are two choices for each of the n objects. So, the total number of ways is $\underbrace{2 \times 2 \times .... \times 2}_{n-\text{times}} = 2^n$.

26. (b) We have, $75600 = 2^4 . 3^3 . 5^2 . 7$

The total number of ways of selecting some or all out of four 2's, three 3's, two 5's and one 7's is (4 + 1) (3 + 1) (2 + 1) (1 + 1) – 1 = 119

But this includes the given number itself. Therefore, the required number of proper factors is 118.

27. (d) The required number of ways

$= (10+1)(9+1)(7+1) - 1 = 879.$

28. (c) The number of ways of selecting 3 persons from 12 people under the given conditon :

Number of ways of arranging 3 people among 9 people seated in a row, so that no two of them are consecutive

= Number of ways of choosing 3 places out of the 10 [8 in between and 2 extremes]

$= {}^{10}C_3 = \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1} = 5 \times 3 \times 8 = 120$

29. (c) $m = (3n)! = 1.2.3.4.5.....(3n)$

= [2 . 4 . 6 .....(2n)] [1 . 3 . 5 . 7]

$= 2^n$ (n !) (1 . 3 . 5 . 7 ......)

⇒ 2^n divides m

Also, m = 3n ! = [3 . 6 . 9 .... (3n)]

[1 . 2 . 4 . 5 .....]

$= 3^n$ (n !) [1 . 2 . 4 . 5 .....]

⇒ 3^n divides m

Since, H.C.F. of $(2^n, 3^n) = 1$, we get

2^n divides m and 3^n divides m

⇒ 6^n divides m

Also, $\dfrac{m}{(n!)^3} = \dfrac{3n!}{n!\ 2n!} \times \dfrac{2n!}{n!\ n!}$

$= ({}^{3n}C_n)({}^{2n}C_n)$

= positive integer = k (say)

$\Rightarrow m = (n!)^3 k$

$\Rightarrow (n!)^3$ divides m.

30. (a) We have $\frac{1}{1!\times 11!}+\frac{1}{3!\times 9!}+\frac{1}{5!\times 7!}$

$+\frac{1}{7!\times 5!}+\frac{1}{9!\times 3!}+\frac{1}{11!\times 1!}$

$$\frac{1}{12!}\left[\frac{12!}{1!\times 11!}+\frac{12!}{3!\times 9!}+\frac{12!}{5!\times 7!}+\frac{12!}{7!\times 5!}+\frac{12!}{9!\times 3!}+\frac{12!}{11!\times 1!}\right]$$

$$= \frac{1}{12!}[{}^{12}C_1 + {}^{12}C_3 + ... + {}^{12}C_{11}]$$

$$= \frac{2^{12-1}}{12!} = \frac{2^{11}}{12!}$$

$$\therefore 2\left(\frac{1}{1!\times 11!}+\frac{1}{3!\times 9!}+\frac{1}{5!\times 7!}\right)=\frac{2^{11}}{12!}$$

$$\Rightarrow \frac{1}{1!\times 11!}+\frac{1}{3!\times 9!}+\frac{1}{5!\times 7!}=\frac{2^{10}}{12!}$$

$$\Rightarrow \frac{2^m}{n!}=\frac{2^{10}}{12!} \Rightarrow m = 10, n = 12$$

Given, ${}^kP_3 = 9240$

$\Rightarrow k(k-1)(k-2) = 9240 = 22 \times 21 \times 20$

$\Rightarrow k = 22$

$\therefore m + n - k = 10 + 12 - 22 = 0$

31. (c) Given, ${}^nC_r + {}^nC_{r-1} + {}^{n+1}C_{r-1} + {}^{n+2}C_{r-1} + ...$

$+ {}^{2n}C_{r-1} = {}^{2n+1}C_{r^2-132}$

$\Rightarrow {}^{n+1}C_r + {}^{n+1}C_{r-1} + ... + {}^{2n}C_{r-1} = {}^{2n+1}C_{r^2-132}$

....

.....

$\Rightarrow {}^{2n}C_r + {}^{2n}C_{r-1} = {}^{2n+1}C_{r^2-132}$

$\Rightarrow {}^{2n+1}C_r = {}^{2n+1}C_{r^2-132}$

$\Rightarrow r^2 - r - 132 = 0 \Rightarrow (r-12)(r+11) = 0$

$\Rightarrow r = 12 \Rightarrow n \geq 12$

So, minimum value of n = 12.

32. (a) Total number of words that can be formed = 10^5. Number of words in which no letter is repeated = ${}^{10}P_5$. So, number of words in which at least one letter is repeated = $10^5 - {}^{10}P_5 = 69760$.

33. (a) If a number is divisible by 3, the sum of the digits in it must be a multiple of 3. The sum of the given six numerals is 0 + 1 + 2 + 3 + 4 + 5 = 15. So to make a five digit number divisible by 3 we can either exclude 0 or 3. If 0 is left out, then 5! = 120 number of ways are possible. If 3 is left out, then the number of ways of making a five digit numbers is 4 × 4! = 96, because 0 cannot be placed in the first place from left, as it will give a number of four digits.

Hence, total number = 120 + 96 = 216.

34. (c) Total number of ways = 6 × 6 × ..... to n times $= 6^n$.

Total number of ways to show only even number = 3 × 3 × ...... to n times $= 3^n$.

$\therefore$ required number of ways $= 6^n - 3^n$.

35. (c) Starting with the letter A, and arranging the other four letters, there are 4! = 24 words. These are the first 24 words. Then starting with G, and arranging A, A, I, and N in different ways, there are $\frac{4!}{2!1!1!} = \frac{24}{2} = 12$ words.

Hence, total 36 words.

Next, the 37th word starts with I. There are 12 words starting with I. This accounts up to the 48th word. The 49th word is NAAGI. The 50th word is NAAIG.

36. (c) At least one black ball can be drawn in the following ways:

(i) one black and two other colour balls

(ii) two black and one other colour balls, and

(iii) all the three black balls

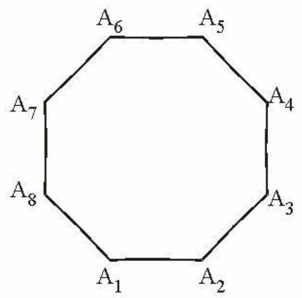
Therefore the required number of ways is

$^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 = 64$.

37. (d) Number of all possible triangles

= Number of selections of 3 points from 8 vertices

$= {}^8C_3 = 56$



Number of triangle with one side common with octagon

$= 8 \times 4 = 32$

(Consider side A_1A_2. Since two points A_3, A_8 are adjacent, 3rd point should be chosen from remaining 4 points.)

Number of triangles having two sides common with octagon : All such triangles have three consecutive vertices, viz., $A_1A_2A_3$, $A_2A_3A_4$, ... $A_8A_1A_2$.

Number of such triangles = 8

$\therefore$ Number of triangles with no side common

$= 56 - 32 - 8 = 16$.

38. (a) No. of ways = coeff. of x^{13} in $(x^2 + x^3 + x^4 + ... + x^9)^3$

[Max coin one can get is 9]

= coeff. of x^{13} in $x^6 (1 + x + x^2 + ... + x^7)^3$

= coeff. of x^7 in $\left(\frac{1-x^8}{1-x}\right)^3$

= coeff. of x^7 in $(1 - x)^{-3}$

= coeff. of x^7 in $(1 + {}^3C_1 x + {}^4C_2 x^2 + ...) = {}^9C_7$

= 36

39. (d) The number is divisible by 9 if sum of digits is divisible by 9. Now, 1 + 2 + 3 +......+ 9 = 45 is divisible by 9, so, seven digit number divisible by 9 should not contain (1, 8) or (2, 7) or (3, 6) or (4, 5) and digits can be arranged in 7! ways.

$\therefore$ No. of such numbers = 4 (7 !)

40. (d) No. of words starting with A are 4 ! = 24

No. of words starting with H are 4 ! = 24

No. of words starting with L are 4 ! = 24

These account for 72 words

Next word is RAHLU and the 74[th] word RAHUL.

41. (c) Required no. of the ways

$= {}^6C_3 \times {}^4C_2 = 20 \times 6 = 120$

42. (d) Two pairs of identical letters can be arranged in

${}^3C_2 \frac{4!}{2!\,2!}$ ways. Two identical letters and two different letters can be arranged in

${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$ ways.

All different letters can be arranged in ${}^8P_4$ ways

$\therefore$ Total no. of arrangements

$= {}^3C_2 \frac{4!}{2!\,2!} + {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} + \frac{8!}{4!} = 2454.$

43. (d) Triangles with vertices on AB, BC and CD are

$3 \times 4 \times 5 = 60$

Triangles with vertices on AB, BC and DA are

$3 \times 4 \times 6 = 72$

Triangles with vertices on AB, CD and DA are

$3 \times 5 \times 6 = 90$

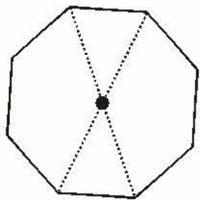
Triangles with vertices on BC, CDand DA are

$4 \times 5 \times 6 = 120$

$\therefore$ Total no. of triangles = 60 + 72 + 90 + 120

= 342

44. (a) A combination of four vertices is equivalent to one interior point of intersection of diagonals.



$\therefore$ No. of interior points of intersection

$= {}^nC_4 = 70$

$\Rightarrow n(n-1)(n-2)(n-3)$

$= 5.\ 6.\ 7.\ 8 \quad \therefore n = 8$

So, number of diagonals $= {}^8C_2 - 8 = 20$

45. (a) Any number greater than a million must be of 7 or more than 7 digits. Here number of given digits is seven, therefoer we have to form numbers of seven digits only.

Now there are seven digits of which 3 occurs thrice and two occurs twice.

$\therefore$ number of numbers formed $= \frac{7!}{2!\,3!} = 420$

But this also includes those numbers of seven digits whose first digit is zero and so in fact they are only six digit numbers.

Number of numbers of seven digits having zero in the first place = 60.

Hence required number = 420 – 60 = 360

46. (d) Required number of numbers = $5 \times 6 \times 6 \times 4$

$= 36 \times 20 = 720.$

47. (c) Required number of numbers

$= 3 \times 5 \times 5 \times 5 = 375$

48. (d) Required numbers are 5! + 5! – 4! = 216.

49. (b) Required sum = (2 + 4 + 6 + ... + 100) + (5 + 10 + 15 + ...+ 100) – (10 + 20 + ... + 100) = 2550 + 1050 – 530 = 3070.

50. (b) No. of ways in which 6 men can be arranged at a round table = (6 - 1)!

Now women can be arranged in 6! ways.

Total Number of ways $= 6! \times 5!$

51. (c) Total number of arrangements of letters in the word GARDEN = 6 ! = 720 there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E.

So, vowels in alphabetical order in $\frac{1}{2} \times 720 = 360$

52. (b) Required number of ways

$= \text{coefficient of } x^2 \text{ in} (x + x^2 + ... x^6)^3$

[$\because$ each box can receive minimum 1 and maximum 6 balls]

$= \text{coeff of } x^8 \text{ in } x^2(1 + x + x^2 + ... + x^5)^3$

$= \text{coeff of } x^5 \text{ in} \left(\frac{1 - x^6}{1 - x}\right)^3$

$= \text{coeff of } x^5 \text{ in } (1 - x)^{-3}$

$= \text{coeff of } x^5 \text{ in } (1 + {}^3C_1 x + {}^4C_2 x^2 + ...)$

$= {}^7C_5 = 21$

53. (c) X - X - X - X - X. The four digits 3, 3, 5,5 can be arranged at (-) places in $\frac{4!}{2!2!} = 6$ ways.

The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways = 10 ways

Total no. of arrangements = $6 \times 10 = 60$ ways

54. (c) Number of elements in the sample space

$= 6 \times 6 = 36$

The sample space is given by

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

. . . .

. . . .

. . . .

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

55. (a) Principal can be appointed in 36 ways.

Vice principal can be appointed in the remaining 35 ways.

Total number of ways = $36 \times 35 = 1260$

56. (c) Required number of ways = ways of selecting 4 objects out of 6 given objects

$= {}^6C_4 = \frac{6 \times 5}{2} = 15$

57. (c) It is a question of arrangement without repetitions.

Required no. of ways = $5 \times 4 \times 3 = 60$

58. (a) Let there be n teams participating in the championship.

Then, total no. of matches = ${}^nC_2 = 153$

or $\frac{n!}{2!(n-2)!} = \frac{1}{2}(n-1) \times n = 153$

or $n^2 - n - 306 = 0$

or $(n - 18)(n + 17) = 0$

or $n = 18$ $[n \neq -ve]$

59. (d) First of all we will prime factorize 8064.

$8064 = 2 \times 4032 = 2^2 \times 2016 = 2^3 \times 1008 = 2^4 \times 504$

$= 2^5 \times 252 = 2^6 \times 126 = 2^7 \times 63$

$= 2^7 \times 3^2 \times 7^1$

Required no. of ways = (7 + 1) (2 + 1) . 1

$= 8 \times 3 = 24$

60. (c) Two particular girls can be arranged in 2! ways and remaining 10 girls can be arranged in 10! ways.

Required no. of ways = $2! \times 10!$

61. (c) 5 letters out of 15 letters can be selected in ${}^{15}C_5$ ways.

62. (b) Let the Vice-chairman & the Chairman form 1 unit. Along with the eight directors, we now have to arrange 9 different units in a circle. This can be done in 8! ways. At the some time, the Vice-chairman & the chairman can be arranged in two different ways. Therefore, the total number of ways = $2 \times 8!$

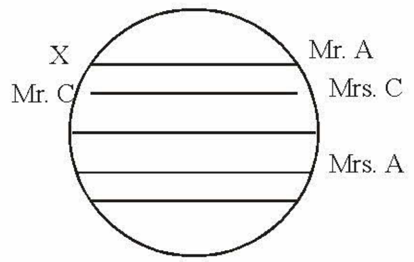
63. (a) Each one of the 26 players played 25 matches and none of the matches ended in a draw Hence all the scores must be even. Also each one of them scored different from the other. The maximum score possible is 50 and minimum score is 0. There are exactly 26 possible scores, 50, 48, 46 .....0.

The ranking is in a alphabetical order means A scored 50, B - 48, Z–0.

This is possible if A wins all the matches B loses only to A win against all others etc.

In final rank, every player win only with all players who are below in final ranking . Since M > N hence M wins over N.

64. (b)



Here we have to find who is sitting at immediate right side of Mr. A i.e. postion X.

From the given conditions, Mrs. A and Mrs. C can't be on the right side of Mr. A. Moreover, Mrs. E can't be on Mr. A's right, as for that Mrs. B has to be on his left instead of Mrs. C. That leaves either of Mrs. B or Mrs. D to fill the concerned position, which is indeed possible without violating the very lax conditions.

65. (c) The no. of ways are

${}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 63$

By option elimination, 2n + 1 = 7. So n = 3.

66. (d) First step- take book 3 to the table B and, second step - put the book 2 on top of 3. Third step - Transfer the arrangement and keep it over book 1 on table A. The last step is transfer the whole arrangement to the table B which is the fourth step to take. Thus total 4 steps are required.

67. (c) Each team will play 7 matches and so any team can win any no. of matches between 0 to 7

0, 1, 2, 3, 4, 5, 6, 7. four team will be selected (7, 6, 5, 3) Thus team which win only 3 matches will be out of the first round.

68. (d) There are 32 black & 32 white square on a chess board then No. of ways in choosing one white & one black square on the chess

$= {}^{32}C_1 \times {}^{32}C_1 = 32 \times 32 = 1024$

No. of ways in which square lies in the same row (white sq. = 4 , black sq = 4, No. of row = 8)

$= {}^4C_1 \times {}^4C_1 \times 8$

= 128

No. of ways in which sq. lie on the same column

$= {}^4C_1 \times {}^4C_1 \times 8 = 128$

total ways in which squares lie on the same row or same column = 128 + 128 = 256

Hence required no. of ways = 1024 - 256 = 768

69. (b) It can be clearly established that the choices (a) , (c) and (d) may or may not be true . Statement (b) can never be true because every person cannot have a different number of acquaintances.

70. (c) Consider the number : x y z where x < y , z < y and $x \neq 0$.

If y = 9 , x can be between 1 to 8 and z can be between 0 to 8

Total combinations = 9 × 8 = 72

If y = 8 , x can be between 1 to 7 and z can be between 0 to 7

combinations = 7 × 8 = 56

Similarly we add all combinations :

8 × 9 + 7 × 8 + 6 × 7 + 5 × 6 + 4 × 5 + 3 × 4 + 2 × 3 + 2 × 1 = 240 ways.

71. (b) As they are consecutively numbered total number of ways will be 6 + 5 + 4 + 3 + 2 + 1 = 21 ways

72. (d) There are 12 points. Since they can be reached from any other point, the edges will be

${}^{12}C_2$ = 66.

Also the number of edges will be maximum 11.

73. (a) Let number of girls = x and the number of boys = y

45 games in which both the players were girls

$\Rightarrow {}^xC_2 = 45$

$\frac{x!}{2!(x-2)!} = x(x-1) = 90 \qquad \therefore x = 10$

190 games, where both the players were boys.

${}^yC_2$ = 190

$\Rightarrow$ y(y – 1) = 380

$\therefore$ y = 20

Hence the total number of games in which one player was a boy and the other was a girl = 10 × 20 = 200

74. (a) For a number to be divisible by 3, the sum of its digits has to be divisible by 3.

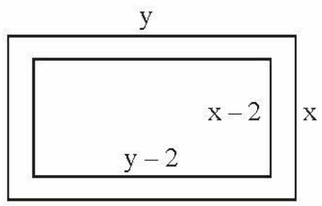
Given : $1000 \le n \le 1200$ ...(1)

Again, for every digit of n to be odd, the four digits can be selected from 1, 3, 5, 7 & 9. Again with (1), the first two digits can be 1 & 1 only. So the sum of the remaining two digits +2 has to be divisible by 3. So the possible digits can be 19, 73, 79, 13 and 55.

These can be organised in 2 + 2 + 2 + 2 + 1 = 9 ways.

75. (c) For each person to know all the secrets the communication has to be between the Englishmen (who knows say E1 French) and one Frenchmen (say F1). The other two in each case will communicate with E1 & F1 respectively. So for minimum no. of calls, E2 gives information to E1 & receives it after E1 interacts with F1. So 2 calls for each of the four E2, E3, F2 and F3, i.e., 8 calls +1 call (between E1 & F1). Hence 9 calls in all.

76. (d)



p + 2 = 1! + (2 × 2!) + (3 × 3!) +.........

+ (10 × 10!) + 2

= (1! + 2) + (2 × 2!) +.........+ (10 × 10!)

= 1 + 2! + (2 × 2!) +.........+ (10 × 10!)

= 1 + 2! (1 + 2) + (3 × 3!) +.........+ (10 × 10!)

= 1 + 3! + (3 × 3!) +.........+ (10 × 10!)

= 1 + 10! + (10 × 10!) = 1 + 11!

Hence p + 2 leaves 1 as remainder when divided by 11!

EXERCISE 2

1. (a) Task 1 can not be assigned to either person 1 or 2 i.e. there are 4 options.

Task 2 can be assigned to 3 or 4

So, there are only 2 options for task 2.

So required no. of ways = 2 options for task 2 × 3 options for task 1 × 4 options for task 3 × 3 options for task 4 × 2 options for task 5 × 1 option for task 6.

= 2 × 3 × 4 × 3 × 2 × 1 = 144

2. (c) Since each question can be selected in 3 ways, by selecting it or by selecting its alternative or by rejecting it. Thus, the total number of ways of dealing with 10 given questions is 3^{10} including a way in which we reject all the questions.

Hence, the number of all possible selections is $3^{10} - 1$.

3. (c) Number of ways of selecting 5 guests from nine friends = ${}^9C_5$

Out of these, ${}^7C_3$ ways are those in which two of the friends occur together [3 more persons to be selected out of remaining 7]

∴ Number of ways, in which two of the friends will not attend the party together

$= {}^9C_5 - {}^7C_3 = 91$.

4. (b) The total number of two factor products = ${}^{200}C_2$. The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore the total number of two factor products which are not multiple of 5 is ${}^{160}C_2$. Hence, the required number of factors which are multiples of 5

$= {}^{200}C_2 - {}^{160}C_2 = 7180$.

5. (c) We have in all 12 points. Since, 3 points are used to form a traingle, therefore the total number of traingles including the triangles formed by collinear points on AB, BC and CA is ${}^{12}C_3 = 220$.

But this includes the following :

The number of traingles formed by 3 points on AB $= {}^3C_3 = 1$

The number of triangles formed by 4 points on BC $= {}^4C_3 = 4$.

The number of triangles formed by 5 points on CA $= {}^5C_3 = 10$.

Hence, required number of traingles

= 220 – (10 + 4 + 1) = 205.

6. (b) There can be 2 possibilities – last digit is odd or even.

Case I : Last digit is odd. Fixing one out of 1, 3 & 5 in the last position. Then only one odd number can occupy odd position which can be chosen in ${}^2C_1$ ways = 2.

One of the two odd digits can be selected for this position in again, ${}^2C_1$ ways = 2.

The other odd number can be put in either of the two even places in 2 ways.

Finally the two even numbers can be arranged in 2! ways.

Hence sum of last digit of these nos.

= (2 × 2 × 2 × 2) (1 + 3 + 5) = 144 ways

Case II : Last digit is even. Then 2 odd nos. out of 3 can be arranged in ${}^3P_2$ = 3! ways.

Again the even nos. can be arranged in 2! ways

∴ Sum = (3! × 2) (2 + 4) = 72 ways.

Total ways = 144 + 72 = 216.

7. (a) Let there be n participants in the beginning. Then the number of games played by (n – 2) players $= {}^{n-2}C_2$

$\therefore {}^{n-2}C_2 + 6 = 84$

(Two players played three games each)

$\Rightarrow {}^{n-2}C_2 = 78 \Rightarrow (n-2)(n-3) = 156$

$\Rightarrow n^2 - 5n - 150 = 0 \Rightarrow n = 15$.

8. (a) The required number of ways

$= {}^{21}C_{10} + {}^{22}C_{10} + {}^{23}C_{10} + ... + {}^{30}C_{10}$

$= ({}^{21}C_{10} + {}^{21}C_{11} + {}^{22}C_{12} + {}^{23}C_{13} + ...$

$+ {}^{30}C_{20}) - {}^{21}C_{10} \quad (\because {}^nC_r = {}^nC_{n-r})$

$= {}^{22}C_{11} + {}^{22}C_{12} + {}^{23}C_{13} + ... + {}^{30}C_{20} - {}^{21}C_{10}$

$(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r)$

....

$= {}^{31}C_{20} - {}^{21}C_{10}$.

9. (c) Let the sides of the game be A and B. Given 5 married couples, i.e., 5 husbands and 5 wives. Now, 2 husbands for two sides A and B can be selected out of 5 = ${}^5C_2$ = 10 ways.

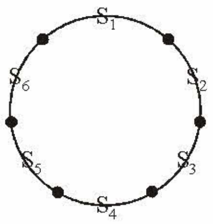
After choosing the two husbands their wives are to be excluded (since no husband and wife play in the same game). So we are to choose 2 wives out of remaining 5 – 2 = 3 wives

i.e., ${}^3C_2$ = 3 ways.

Again two wives can interchange their sides A and B in 2! = 2 ways.

By the principle of multiplication, the required number of ways = 10 × 3 × 2 = 60

10. (b)



Six students S_1, S_2, ..., S_6 can be arranged round a circular table in 5 ! ways. Among these 6 students there are six vacant places, shown by dots (•) in which six teachers can sit in 6 ! ways.

Hence, number of arrangement = 5 ! × 6 !

11. (a) Number of ways of arranging p big animals into m – n big cages = ${}^{m-n}P_p$.

Now remaining animals can be arranged in any cage in ${}^{m-p}P_{m-p}$ ways

∴ Desired number of ways = ${}^{m-n}P_p \times {}^{m-p}P_{m-p}$

12. (d) Two possibilities are there :

(i) Chemistry part I is available in 8 books with Chemistry part II.

or

(ii) Chemistry part II is available in 8 books but Chemistry part I is not available.

Total No. of ways

$= 1\times {}^6C_1 + {}^7C_3$

$= 6+\dfrac{7\times 6\times 5}{3\times 2} = 6+35 = 41$

13. (a) As all the points are equally spaced, the area of all the convex pentagons will be the same.

14. (a) No. of words which have at least one letter repeated = total no. of words – total no. of words in which no letter is repeated

$= 10^5 - {}^{10}P_5$

= 100000 – 10 × 9 × 8 × 7 × 6

= 100000 – 30240 = 69760

15. (b) Under the given restrictions, 5 questions can be selected in the following ways :

2 questions from the first section and 3 questions from the second section

OR

3 questions from the first section and 2 questions from the second section.

Required no. of ways

$= {}^4C_2 \times {}^4C_3 + {}^4C_3 \times {}^4C_2$

= 24 + 24 = 48

16. (b) Six balls can be selected in the following ways: one red balls and 5 blue balls or

Two red balls and 4 blue balls

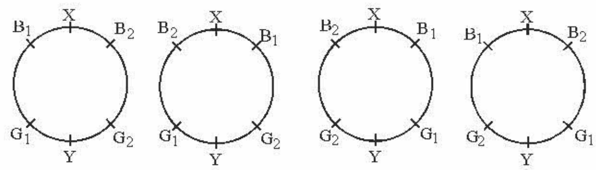
Total number of ways

$= {}^3C_1 \times {}^7C_5 + {}^3C_2 \times {}^7C_4$

$= 3\times\dfrac{7\times 6}{2\times 1} + 3\times\dfrac{7\times 6\times 5}{3\times 2\times 1}$

= 63 + 105 = 168

17. (c) Four possible arrangements are :



18. (b) Six students can be arranged in a row is 6! ways. Another six students can be further arranged in 6! ways.

Hence, total number of ways = 6! × 6!

Note : Do not get confused with the two type of booklets. The booklets can be distributed in 2 ways.

Both these arrangements will be part of the permutation of students arrangement.

1	2	1	2	1	2
1	2	1	2	1	2

2	1	2	1	2	1
2	1	2	1	2	1

19. (a) Two tallest boys can be arranged in 2! ways. Rest 18 can be arranged in 18! ways.

Girls can be arranged in 6! ways.

Total number of ways of arrangement

$= 2! \times 18! \times 6!$

$= 18! \times 2 \times 720 = 18! \times 1440$

20. (d) To construct 2 roads, three towns can be selected out of 4 in $4 \times 3 \times 2 = 24$ ways. Now if the third road goes from the third town to the first town, a triangle is formed, and if it goes to the fourth town, a triangle is not formed. So there are 24 ways to form a triangle and 24 ways of avoiding a triangle.

21. (d) Each box can be filled in 2 ways.

Hence total no. of ways $= 2^5 = 32$

Blue balls can not be filled in adjacent boxes

Total no. of such cases in which blue is filled in 2 adjacent boxes is

2 blue + 3 blue + 4 blue + 5 blue

= 4 ways (12, 23, 34, 45) + 3 ways (123, 234, 345)

+ 2 ways (1234, 2345)+ 1 way

= 10 ways

Hence total cases in which blue balls can not be filled in adjacent boxes = 32 - 10 = 22

22. (b) Out of five girls, he has to invite exactly 3. This can be done in ${}^5C_3$ ways. Out of 4 boys he may invite either one or two or three or four or none of them. According to the standard formula, this may be done in 2^4 ways. Hence, the total number of ways in which he can invite his friends are ${}^5C_3 \times 2^4 = 10 \times 16 = 160$ ways