CH 1 NUMBER SYSTEM

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (c) From the given alternatives,

   $112 \times 114 = 12768$

   $\therefore$ Large number = 114

2. (d) Amount received by each person

   $= ₹\ \frac{50176}{32} = 1568$

3. (d) Amount received by each person

   $= \frac{172850}{25} = ₹\ 6914$

4. (a) Total number of candles = $15 \times 12 \times 39 = 7020$

5. (a) Out of the given alternatives,

   $137 \times 139 = 19043$

   $\therefore$ Required smaller number = 137

6. (d) Let the number be x and (x + 1),

   $\therefore\ x(x + 1) = 8556$

   or, $x^2 + x - 8556 = 0$

   or, $x^2 + 93x - 92x - 8556 = 0$

   or, $(x^2 + 93)(x - 92) = 0$

   $\therefore\ x = 92$

7. (c) Quantity of wheat for 7 days = 112 kg

   $\therefore$ Quantity of wheat for 1 day = $\frac{112}{7}$ kg

   $\therefore$ Quantity of wheat for 69 days

   $= \frac{112}{7} \times 69 = 1104$ kg

8. (a) Required amount $= \frac{41910}{22} = ₹1905$

9. (c) Let the smaller number be x

   $\because\ x \times (x + 2) = 4488$

   $\Rightarrow x^2 + 2x - 4488 = 0$

   $\Rightarrow (x + 68)(x - 66) = 0$

   $\therefore\ x = 66$

10. (a) Required number of bananas

    $= \frac{21}{7} \times 54 = 162$ dozen

11. (d) Amount received by each person

    $= \frac{72128}{46} = ₹1568$

12. (b) $38^2 = 1444$

    $39^2 = 1521$

    $\therefore$ Required number = $1521 - 1500 = 21$

13. (e) Let the three consecutive integers be x, x + 1 and x + 2

    According to the question,

    $x + x + 1 + x + 2 = 39$

    or, $3x + 3 = 39$

    or, $3x = 39 - 3 = 36$

    or, $x = \frac{36}{3} = 12$

    $\therefore$ Required largest number

    $= x + 2 = 12 + 2 = 14$

14. (d) Number of pieces $= \frac{455.8}{8.6} = 53$

15. (d) Out of the given alternatives,

$56 \times 57 = 3192$

16. (c) Amount received by each student

$= \frac{15487}{76} = \approx ₹204$

17. (b) Quicker Approach:

The unit's digit of the number 16128 is 8,

From the given answer choices,

$126 \times 128 = 16128$

∴ Required larger number = 128

18. (c) Number of mangoes = 12 dozens

$= 12 \times 12 = 144$

∴ Number of mangoes in 43 boxes

$= 43 \times 144 = 6192$

19. (b) Requirement of bananas for 1 day in the canteen = 13 dozens

∴ Requirement of bananas for 9 weeks i.e. 63 days

$= 63 \times 13$ dozens

$= 63 \times 13 \times 12 = 9828.$

20. (e) Let the cost of one chair be ₹ x and that of a table be

= ₹y

According to the question,

$3x + 10y = ₹9856$

or, $2 \times (3x + 10y) = 2 \times 9856$

∴ $6x + 20y = ₹19712$

21. (a) Amount received by each person

$= \frac{123098}{61} = ₹2018$

22. (a) According to the question,

$x + x + 2 + x + 4 + x + 6 + x + 8 = 140$

or, $5x + 20 = 140$

or, $5x = 120$

∴ $x = \frac{120}{5} = 24$

∴ $x + 8 = 24 + 8 = 32$

The next set of five consecutive even number will start with = 34

∴ Required sum

$= 34 + 36 + 38 + 40 + 42 = 190$

23. (d) Let the cost of a table be = ₹x and that chair be = ₹y

According to the question,

$5x + 6y = ₹2884$

∴ $3 \times 5x + 3 \times 6y = 3 \times ₹2884$

or, $15x + 18y = ₹8652$

24. (a) From the given alternatives,

$1763 = 43 \times 41$

25. (e) Required quantity of rice

$= \frac{4560 \times 7}{30}$ kg = 1064 kg

26. (d) amount received by each person

$= \frac{13957}{45} = ₹310.15 \approx ₹\ 310$

27. (d) Let the smaller number be x

∵ $x(x + 2) = 5358$

⇒ $x^2 + 2x - 5328 = 0$

⇒ $(x + 74)(x - 42) = 0$

∴ $x = 72$

28. (c) $\frac{x + x + 2 + x + 4 + x + 6}{4} = 27$

$\Rightarrow x = \frac{27 \times 4 - 12}{4}$

$= \frac{96}{4} = 24$

∴ Highest number = 24 + 6 = 30

29. (a) Decimal equivalents of given fractions:

$\frac{1}{2} = 0.5;$ $\frac{2}{3} = 0.67;$

$\frac{5}{9} = 0.56;$ $\frac{6}{13} = 0.46;$

$\frac{7}{9} = 0.78$

∴ 0.46 < 0.5 < 0.56 < 0.67 < 0.78

$\frac{6}{13} < \frac{1}{2} < \frac{5}{9} < \frac{2}{3} < \frac{7}{9}$

∴ Fourth fraction = $\frac{2}{3}$

30. (a) Decimal equivalents of fractions

$\frac{7}{8} = 0.875,$ $\frac{4}{5} = 0.8$

$\frac{8}{14} = 0.57,$ $\frac{3}{5} = 0.6$

$\frac{5}{6} = 0.83$

∴ 0.875 > 0.83 > 0.8 > 0.6 > 0.57

∴ $\frac{7}{8} > \frac{5}{6} > \frac{4}{5} > \frac{3}{5} > \frac{8}{14}$

31. (b) Decimal equivalent of given fractions:

$\frac{2}{5} = 0.4;$ $\frac{3}{4} = 0.75;$ $\frac{4}{5} = 0.8;$

$\frac{5}{7} = 0.714;$ $\frac{6}{11} = 0.545$

Clearely, 0.4 < 0.545 < 0.714 < 0.75 < 0.8

∴ $\frac{2}{5} < \frac{6}{11} < \frac{5}{7} < \frac{3}{4} < \frac{4}{5}$

32. (a) Let the larger and smaller numbers be x and y respectively.

Then, x – y = 3 ...(i)

and, $x^2 - y^2 = 63$

⇒ (x + y) (x – y) = 63

⇒ $(x + y) = \frac{63}{3} = 21$ ...(ii)

From equation (i) and (ii),

x = 12

33. (a) Let the number be = x

According to the question,

$x - \frac{2x}{5} = 30$

⇒ $\frac{3x}{5} = 30$

⇒ $x = \frac{30 \times 5}{3} = 50$

34. (c) Sum to be collected from 54 students = 60×54 = ₹3240

Sum collected from 45 students = 60 × 45 = ₹2700

Difference = 3240 - 2700 = ₹540

∴ Additional amount to be paid by each student

$= \frac{540}{45} = ₹12$

35. (d) Let the number be x.

∵ x^2 - (12)3 = 976

∴ x^2 = 976 + 1728 = 2704

∴ x = $\sqrt{2704} = 52$

36. (c) ∵ 5 chairs + 8 tables = ₹6574

∴ 10 chairs + 16 tables = 6574 × 2 = ₹ 13148

37. (b) Let the number be *x*.

$\because\ x^2 + (56)^2 = 4985$

$\Rightarrow x^2 = 4985 - 3136 = 1849$

$\therefore\ x = \sqrt{1849} = 43$

38. (c) $\left(1-\frac{1}{5}\right)$ of the number = 84

$\therefore$ number $= \left(\frac{84\times5}{4}\right) = 105$

39. (e) $\frac{3}{5} = 0.6, \frac{1}{8} = 0.125,$

$\frac{8}{11} = 0.727, \frac{4}{9} = 0.44,$

$\frac{2}{7} = 0.285, \frac{5}{7} = 0.714,$

$\frac{5}{12} = 0.416$

Descending order :

$\frac{8}{11}, \frac{5}{7}, \frac{3}{5}, \frac{4}{9}, \frac{5}{12}, \frac{2}{7}, \frac{1}{8}$

So, $\frac{3}{5}$ is the third.

40. (c) Let Farah's age at the time of her marriage be x.

Then, $(x+8) = x\times\frac{9}{7}$

$\Rightarrow \frac{9x}{7} - x = 8$

$\Rightarrow x = \frac{8\times7}{2} = 28$ years

$\therefore$ Farah's present age = 28 + 8 = 36 years

$\therefore$ Daughter's age 3 years ago $= 36\times\frac{1}{6} - 3$

= 3 years

41. (d) A + C = 146

or A + A + 4 = 146

or A $= \frac{146-4}{2} = 71$

$\therefore$ E = A + 8 = 71 + 8 = 79

42. (b) Let the numbers be x and $(x + 2)$

Then, x × (x + 2) = 582168

$\Rightarrow x^2 + 2x - 582168 = 0$

$\Rightarrow x^2 + 764x - 762x - 582168 = 0$

$\Rightarrow (x + 764)(x - 762) = 0$

$\Rightarrow x = 762$

43. (d) Let the two numbers be x and (x + 2).

Then, $x^2 + (x + 2)^2 = 6500$

$\Rightarrow x^2 + x^2 + 4x + 4 = 6500$

$\Rightarrow 2x^2 + 4x - 6496 = 0$

$\Rightarrow x^2 + 2x - 3248 = 0$

$\Rightarrow x^2 + 58x - 56x - 3248 = 0$

$\Rightarrow (x + 58)(x - 56) = 0$

$\Rightarrow x = 56$

EXERCISE 2

1. (a) Let the two-digit number be = 10x + y, where x > y

According to the question,

10x + y – 10y – x = 18

or, 9x – 9y = 18

or, 9(x – y) = 18

or, $x - y = \frac{18}{9} = 2$ ...(i)

and, x + y = 12 ...(ii)

From equations (i) and (ii)

$2x = 14 \Rightarrow x = \frac{14}{2} = 7$

From equation (i)

$y = 7 - 2 = 5$

$\therefore$ Required product = xy = $7 \times 5 = 35$

2. (a) Let the two-digit number be = 10x + y

where x > y

According to the question,

$10x + y - 10y - x = 27$

or, $9x - 9y = 27$

or, $9(x - y) = 27$

$\therefore$ $x - y = 3$ ...(i)

and $x + y = 15$ ...(ii)

Solving these two equations, we get, x = 9, y = 6

$\therefore$ Required product = $9 \times 6 = 54$

3. (a) A hen has two legs whereas a cow has four legs. But both of them have one head each.

Let Kishan have x cows

$\therefore$ Number of hens = 59 – x.

According to the question,

$4 \times x + (59 - x) \times 2 = 190$

or, $4x + 118 - 2x = 190$

or, $2x = 190 - 118 = 72$

$\therefore$ $x = \frac{72}{2} = 36$

Number of cows = 36

4. (c) Let the number of hens = x

$\therefore$ Number of goats = 43 – x

According to the question,

$x \times 2 + (43 - x) \times 4 = 142$

or, $2x + 172 - 4x = 142$

or, $2x = 172 - 142$

$\therefore$ $x = \frac{30}{2} = 15$

$\therefore$ Number of hens = 15

5. (c) Let the two-digit number be

= 10 x + y, where x < y.

Number obtianed after interchanging the digits

= 10 y + x

According to the question,

$10y + x - 10x - y = 9$

or, $9y - 9x = 9$

or, $9(y - x) = 9$

or, $y - x = 1$ ...(i)

and $x + y = 15$ ...(ii)

From equations (i) and (ii),

y = 8 and x = 7

$\therefore$ Required product = $8 \times 7 = 56$

6. (a) Let the number be (10x + y)

Then, $(10x + y) - (10y + x) = 18$

$\Rightarrow$ $9x - 9y = 18$

$\Rightarrow$ $x - y = 2$ ...(i)

and, $x + y = 16$ ...(ii)

x = 9, y = 7

From equations (i) and (ii),

So, the number is $(10 \times 9 + 7) = 97$

7. (b) Let Seema's present age be x years.

Then, Seema's son's present age = $\frac{x}{4}$ years

Seema's father's present age = $\frac{7x}{4}$ years.

Then, $x + \frac{x}{4} + \frac{7x}{4} = 32 \times 3$

$\Rightarrow$ $12x = 96 \times 4$

$\Rightarrow$ $x = \frac{96 \times 4}{12} = 32$

$\therefore$ Required difference = $\frac{7 \times 32}{4} - \frac{32}{4}$

= 56 – 8 = 48 years

8. (d) Lowest number of set A = $\frac{280}{5} - 4$ = 52

Lowest number of other set = 52 × 2 – 71 = 33

∴ Required sum = 33 + 34 + 35 + 36 + 37

= 175

9. (c) Let total number of goats be x.

Then, total number of hens = (90 – x)

So, x × 4 + (90 – x) × 2 = 248

⇒ 4x – 2x = 248 – 180

$x = \frac{68}{2} = 34$

10. (e) Let the two digits be x and y.

Then, x + y = 15 ... (i)

x – y = 3 ... (ii)

from equation (i) and (ii), x = 9, y = 6

∴ Product = 9 × 6 = 54

11. (b) Divisor = [Sum of remainders]

– [Remainder when sum is divided]

= 11 + 21 – 4 = 28

12. (c) Let number be N.

Then, N = Divisor × Q_1 + 23

2N = Divisor × Q_2 + 11, where Q_1 and Q_2 are quotients respectively.

13. (d) Let the number be 5q + 3, where q is quotient

Now, $(5q+3)^2 = 25q^2 + 30q + 9$

$= 25q^2 + 30q + 5 + 4$

$= 5[5q^2 + 6q + 1] + 4$

Hence, remainder is 4

14. (d) Let us divide the different powers of 4 by 6 and find the remainder.

So remainder for $4^1 = 4$, $4^2 = 4$, $4^3 = 4$, $4^4 = 4$, $4^5 = 4$, $4^6 = 4$ and so on.

Hence remainder for any power of 4 will be 4 only.

15. (b) $\frac{7^{84}}{342} = \frac{(7^3)^{28}}{342} = \frac{(343)^{28}}{342}$

By remainder theorem, $\frac{(343)^{28}}{342}$ will have the same remainder as $\frac{1^{28}}{342}$ i.e. the remainder is 1.

Alternative :

$\frac{(343)^{28}}{342} = \frac{(342+1)^{28}}{342}$

$= \frac{(342)^{28} + {}^{28}C_1(342)^{27} + ... + {}^{28}C_{27}342 + 1}{342}$

Clearly, 1 is the remainder.

16. (a) ∵ 56 = $d_1 \times d_2$

∴ required remainder = $d_1r_2 + r_1$ where d_1 = 7 and r_1 = 3 and r_2 = 5

17. (b) He wants to write from 1 to 999. He has to write 9 numbers of one digit, 90 numbers of two digits and 900 numbers of three digits.

Total number of times = 1 × 9 + 2 × 90 + 3 × 900 = 2889

18. (a) ∴ Complete remainder = $d_1d_2r_3 + d_1r_2 + r$ = 3 × 5 ×4 + 3 × 2 + 1 = 67

Divided 67 by 8, 5 and 3, the remainders are 3, 3, 1.

19. (a) The required numbers are 307, 317, 327, 337, 347, 357, 367, 370, 371, 372, 373, 374, 375, 376, 378, 379, 387, 397.

Hence there are 18 numbers.

20. (b) Here, number of integers next higher and next lower are same (=4).

Now, since 81 is divisible by 9, therefore, the sum is divisible by 9

21. (a) Let the original number is x. Then

$\frac{2}{3}x = x - 30$

$\Rightarrow x\left(1-\frac{2}{3}\right)=30 \Rightarrow x=90$

22. (b) $\therefore \frac{500}{13}=38$

23. (c) A number is divisible by 9 if the sum of its digits is divisible by 9.

Here 5 + 4 + 3 + 2 + * + 7 = 21 + *

So, the digit in place of * is 6

24. (a) Sum of the digits of the 'super' number

= 1 + 2 + 3 + ... + 29

$=\frac{29}{2}.\{2\times1+(29-1).1\}$

$=\frac{29}{2}.(2+28)=\frac{29\times30}{2}=29\times15=435$

Now, sum of digits in the number 435 = 4 + 3 + 5 = 12 which gives a remainder of 3 when divided by 9.

25. (a) Here 44 = 11 × 4

∴ the number must be divisible by 4 and 11 respectively. Test of 4 says that 9y must be divisible by 4 and since y > 5, so y = 6

Again , x 9596 is divisible by 11, so x + 5 + 6 = 9 + 9

⇒ x = 7

Thus x = 7, y = 6

26 (a) 24162 = 89x + 43

⇒ x = (24162 – 43) ÷ 89 = 271

27. (c) By actual division, we find that 999999 is exactly divisible by 13. The quotient 76923 is the required number.

28. (d) Complete remainder = $d_1d_2r_3 + d_1r_2 + r_1$

= 5 × 6 × 7 + 5 ×4 + 3 = 233.

Dividing 233, by reversing the divisors i.e. by 8, 6, 5; respective remainders are 1, 5, 4.

29. (b) By division Algorithm,

49471 = 246 × D + 25

⇒ D = 201

30. (a) Let the number be z. Now 385 = 5 × 7 ×11

5	z	Remainders
7	y	4
11	x	6
	102	10

x = 11 × 102 + 10 = 1132

y = 7x + 6 = 7 × 1132 + 6 = 7930

z = 5y + 4 = 5 × 7930 + 4 = 39654

31. (b) Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.

Now, 7^4 gives unit digit 1.

∴ 7^{153} gives unit digit (1 × 7) = 7. Also 1^{72} gives unit digit 1.

Hence, unit's digit in the product = (7 × 1) = 7.

32. (a) Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

33. (c) On dividing 803642 by 11, we get remainder = 4.

∴ Required number to be added = (11 – 4) = 7.

34. (a) Number = (296 × Q) + 75 = (37 × 8Q) + (37 × 2) + 1

= 37 × (8Q + 2) + 1.

∴ Remainder = 1.

35. (a)

4	x	Remainders
5	y	–2
6	z	–3
	1	–4

z = 6 × 1 + 4 = 10

y = 5 × 10 + 3 = 53

$x = 4 \times 53 + 2 = 214$

36. (c) On dividing 6709 by 9, we get remainder = 4.

$\therefore$ Required number to be subtracted = 4.

37. (a) On dividing 427398 by 15, we get remainder = 3.

$\therefore$ Required number to be subtracted = 3.

38. (c) Unit digit in $(3127)^{173}$ = Unit digit in $(7)^{173}$. Now, 7^4 gives unit digit 1.

$\therefore$ $(7)^{173} = (7^4)^{43} \times 7^1$. Thus, $(7)^{173}$ gives unit digit 7.

39. (d) Number = $(31 \times Q) + 29$. Given data is inadequate.

40. (c) Unit digit in 7^4 is 1.

Unit digit in 7^{68} is 1.

$\therefore$ Unit digit in $7^{71} = 1 \times 7^3 = 3$

Again, every power of 6 will give unit digit 6.

$\therefore$ Unit digit in 6^{59} is 6.

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.

$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$

= Unit digit in $(3 \times 6 \times 3) = 4$.

41. (c) Let x be the number of times, then

$79x + 43759 = 50{,}000$

$\Rightarrow x = (50000 - 43759) \div 79 = 79$

42 (b)

$$7^{84}/342 = (7^3)^{28}/(7^3-1) = ((7^3)^{28}-1+1)/(7^3-1)$$

$$= ((7^3)^{28}-1)/(7^3-1) + 1/(7^3-1)$$

$((7^3)^{28} - 1) / (7^3 - 1)$ is always divisible as it is in the form of $(x^n - y^n) / (x - y)$, hence the remainder is 1.

43. (b) Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.

$\therefore$ Unit digit in 7^{95} is 3.

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{56} is 1.

$\therefore$ Unit digit in 3^{58} is 9.

$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

44 (b) **The digit in the unit's place of 2^{51}** is equal to the remainder when 2^{51} is divided by 10. $2^5 = 32$ leaves the remainder 2 when divided by 10. Then $2^{50} = (2^5)^{10}$ leaves the remainder $2^{10} = (2^5)^2$ which in turn leaves the remainder $2^2 = 4$. Then $2^{51} = 2^{50} \times 2$, when divided by 10, leaves the remainder $4 \times 2 = 8$.

45. (c) $55^3 + 17^3 - 72^3 = (55)^3 + (17)^3 - (55+17)^3$

$$= 55^3 + 17^3 - (55)^3(17)^3 - 3 \times 55 \times 17 \times 72$$

$$= -3 \times 55 \times 17 \times 72$$

46. (b) The number is divisible by 18 i.e., it has to be divisible by 2 and 9.

$\therefore$ B may be 0, 2, 4, 6, 8.

A + 4 + 5 + 7 + 1 + 2 + 0 + 3 + B

= A + B + 22.

A + B could be 5, 14 (as the sum can't exceed 18, since A and B are each less than 10).

So, A and B can take the values of 6, 8.

47. (b) Number is of the form = 7n + 3; n = 1 to 13

$$\text{So, } S = \sum_{n=1}^{13}(7n+3) = 7 \times 13 \times 7 + 39 = 676$$

48. (a) a, a + 2, a + 4 are prime numbers.

Put value of 'a' starting from 3, we will have 3, 5 and 7 as the only set of prime numbers satisfying the given relationships.

49. (c) The expression becomes $(19 - 4)^{23} + (19 + 4)^{23}$. All the terms except the last one contains 19 and the last terms get cancelled out. Hence the remainder obtained on dividing by 19 will be 0.

Alternatively : $a^n + b^n$ is always divisible by (a + b), if n is odd

Here n is odd (23).

So the given expression is divisible by 15 + 23 = 38, which is a multiple of 19.

50. (a) Remember that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$x = (16^3 + 17^3 + 18^3 + 19^3)$$

$x = (16^3 + 19^3) + (17^3 + 18^3)$

$x = (16 + 19)(16^2 + 19^2 - 16 \times 19) + (17 + 18)$

$(17^2 + 18^2 - 17 \times 18)$

$x = 35[16^2 + 19^2 - 16 \times 19 + 17^2 + 18^2 - 17 \times 18]$

$x = 35 \times$ (Even number)

Hence, x is divisible by 70 and leaves remainder as zero.

51. (a) The number 30^{2720} will have 2720, zero's.

For the right most non-zero digit we have to check the power cycle of 3 and find when their multiplication again leads to a 3 as the right most digit.

$3^1 = 3; 3^2 = 9; 3^3 = 27; 3^4 = 81; 3^5 = 243$

Hence, 3 will appear after every fourth power of 3.

Hence, $30^{2720} = 3^{2720} \times 10^{2720} = (3^4)^{680} \times 10^{2720}$

As the number 2720 is an exact multiple of 4, hence the last digit will be 1 similar to what we find in 3^4.

52. (d) As $x^n - y^n$ is divisible by $x - y$ if n is odd.

$x^n - y^n = (x - y)(x^{n-1}y^0 + x^{n-2}y^1 + ...+ x^0y^{n-1})$

Hence numerator becomes

$= (30 - 29)(30^{64} + ...+ 29^{64})$

$= 30^{64} + ...+ 29^{64}$

$\therefore R = \frac{30^{64} + ...+ 29^{64}}{30^{64} + 29^{64}}$

Clearly the numerator is greater than the denominator. Hence R > 1.0

53. (b) Putting the value of x = – 0.5 in all the options.

(a) $2^{1/-0.5} = 2^{-2} = \frac{1}{4}$ (b) $\frac{1}{-0.5} = -2$

(c) $\frac{1}{(-0.5)^2} = 4$ (d) $2^{-0.5} = \frac{1}{\sqrt{2}}$

So, clearly (b) is smallest.

54. (b) In this question it is advisable to raise all the numbers to the power of 12, so the numbers become,

$(2^{1/2})^{12}, (3^{1/3})^{12}, (4^{1/4})^{12}, (6^{1/6})^{12}$

or $2^6, 3^4, 4^3, 6^2$ or 64, 81, 64, 36

So, $3^{1/3}$ is the largest.

55. (c) $7^0 = 01$

$(7)^1 = 07$

$(7)^2 = 49$

$(7)^3 = 243$

$(7)^4 = 2401$

$(7)^5 = 16807$

$(7)^6 = 117649$

$(7)^7 = 823543$

$(7)^8 = 5764801$

Here we see last two digit 01 is repeated when power of $(7)^0$ is increased by 4 each time.

Now 2008 ÷ 4 = 502

Hence when power of $(7)^0$ increases 502 times by 4 (each time), then we get that 01 is the last two digits in the number $(7)^{2008}$.

56. (b) Let the common remainder be x. Then numbers (34041 – x) and (32506 – x) would be completely divisible by n.

Hence the difference of the numbers (34041 – x) and (32506 – x) will also be divisible by n

or (34041 – x – 32506 + x) = 1535 will also be divisible by n.

Now, using options we find that 1535 is divisible by 307.

57. (c) $N = 55^3 + 17^3 - 72^3 = (54 + 1)^3 + (18 - 1)^3 - 72^3$

or $N = (51 + 4)^3 + 17^3 - (68 + 4)^3$

These two different forms of given expression is divisible by 3 and 17 both.

58. (b) Required Divisor = (sum of remainders)

– Remainder when sum is divided

= [4375 + 2986] – 2361 = 5000

59. (c) 987 = 3 × 7 ×47

So, required number must be divisible by each one of 3, 7, 47.

None of the numbers in (a) and (b) are divisible by 3, while (d) is not divisible by7.

∴ Correct answer is (c).

60. (c) Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, and 13. as we may write, 222222 = 2 × 111111, 333333 = 3 × 111111, etc.

61. (b) Complete remainder = $d_1r_2 + r_1$

= 4 × 4 + 1 = 17

Now, 17 when divided successively by 5 and 4

∴ The remainders are 2, 3.

62. (d) The required no. is 3[4 (7x + 4) + 1] + 2 = 84x + 53

So the remainder is 53, when divided by 84.

63. (d) Given a = 6b = 12c = 27d = 36e

Multiplied and Divide by 108 in whole expression

$$\frac{108a}{108} = \frac{108b}{18} = \frac{108c}{9} = \frac{108d}{4} = \frac{108e}{3}$$

$$\frac{1}{108}a = \frac{1}{18}b = \frac{1}{9}c = \frac{1}{4}d = \frac{1}{3}e = 1 \text{ (say)}$$

⇒ a = 108, b = 18, c = 9, d = 4, e = 3

So it is clear that $\left(\frac{a}{6}, \frac{c}{d}\right)$ contains a number

$\frac{c}{d} = \left(\frac{9}{4}\right)$ which is not an integer

64. (c) $N = 1421 \times 1423 \times 1425$, when these numbers are divided by 12 we have remainders as 5, 7, 9.

The product of remainders when divided by 12 gives 3 as its remainder. Thus when N divided by 12 remainder is 3

EXERCISE 3

1. (a) Let total number of seats in the stadium be p;

number of seats in the lower deck be x and number of seats in upper deck be y.

∴ p = x + y, x = p/4, y = 3p/4

Now in the lower deck, 4x/5 seats were sold and x/5 seats were unsold.

No. of total seats sold in the stadium = 2p/3.

No. of unsold seats in the lower deck = x/5 = p/20

No. of unsold seats in the stadium = p/3

∴ Required fraction = $\frac{p/20}{p/3} = \frac{3}{20}$

2. (d) x is prime say 7

y is not prime but composite no. say 8, 9, 21

(a) 9 – 7 = 2 (b) 7 × 8 = 56 (c) $\frac{21+7}{7} = 4$

Put x = 2 and y = 6 and check for the options.

By hit and trial all the 3 options can be proved wrong

3. (d) Let n = 6

Therefore $\sqrt{n} = \sqrt{6} \approx 2.4$

Now, the divisor of 6 are 1, 2, 3

If we take 2 as divisor then $\sqrt{n} > 2 > 1$.

Statement I is true.

If we take 3 as divisor then $6 > 3 > 2.4$, i.e. $n > \sqrt{n}$

Therefore statement II is true

4. (b) Let the 3 digits of number A be x, y and z

Hence A = 100x + 10y + z

On reversing the digits of number A, we get the number B i.e., z y x.

$\therefore$ B = 100z + 10y + x

As B > A $\Rightarrow$ z > x ...(i)

B – A = 99z – 99x = 99(z – x)

As 99 is not divisible by 7

so (z – x) has to be divisible by 7. ...(ii)

Using (i) & (ii), the only possible values of z and x are (8, 1) and (9, 2)

So the minimum and maximum range of A are 108 and 299, which $\in 106 < A < 305$

5. (d) The no. can be 2 or 3 digit.

Firstly let n be the two digit no.

Therefore, n = 10x + y

$p_n + s_n = n \Rightarrow xy + x + y = 10x + y \Rightarrow xy - 9x = 0$

$\Rightarrow y = 9$ as $x \neq 0$

So the numbers can be 19, 29.........99, i.e., 9 values.

For 3 digits n = 100x + 10y + z

$\Rightarrow xyz + x + y + z = 100x + 10y + z$

$\Rightarrow xyz = 99x + 9y$ or $xz = \frac{9(11x+y)}{y}$

It can be verified using various values of y that this equation do not have any solution.

E.g. : For y = 9, x(z – 11) = y which is not possible. So in all 9 integers.

6. (a) Since in the four digits number first two digits are equal and the last two digits are also equal, therefore we can suppose that the digit at the thousand and hundred place each be *x* and the digit at the tenth and unit place each be *y*.

Hence, the four digits number

$= 1000x + 100x + 10y + y = 11(100x + y)$

This number $11(100x + y)$ will be perfect square, if $100x + y$ is of the form $11n$, where n is a perfect square

Now $100x + y = 11n \Rightarrow y = 11n - 100x$

On checking, we get for the value $n = 64$ (a perfect square) only, $y = 704 - 100x$, for which a single digit positive integral value 7 of x, the value of $y = 4$, which is the single digit positive integer.

There is no single digit positive integral value of y for any other single positive integral value of x for the equation $y = 704 - 100x$

Hence, 7744 is the only for digits number.

7. (a) $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$

$\Rightarrow 12n + 48m - mn - 576 = -576$

$$m - 12 = \frac{576}{n - 48} \quad ...(i)$$

Since n is an odd, therefore, $(n - 48)$ is an odd.

Also – 576 is an even, therefore $(m - 12)$ is definitely even.

Now n is an odd integer less than 60. Hence, on checking, we get all possible value of n are 49, 51 and 57.

Therefore, there are three value of n

8. (c) $1 + 2 + 3 + ... + 40 = = \frac{40 \times 41}{2} = 820$

Since at each time any two numbers *a* and *b* are erased and a single new number (a + b – 1) is writen. Hence, each one is subtracted and this process is repeated 39 times. Therefore, number left on the board at the end = 820 – 39 = 781.

9. (d) All the numbers greater than 999 but not greater than 4000 are four digits number.

The number of numbers between 999 and 4000

$= 3 \times 5 \times 5 \times 5 = 375$

Since one number 4000 will also be included. Hence number of toal number greater than 999 but not greater than 4000 = 375 + 1 = 376

10. (c) Let the total no. of males in Chota Hazri be x.

According to the question,

No. of female in Chota Hazri = 2x

Village	Male	Female
Chota Hazri	x	$2x$
Mota Hazri	$x+4522$	$x+8542$

According to the question,

$2x+2910=x+8542 \Rightarrow x = 5632$

11. (b) Suppose D = 24 $\therefore$ $S = (2 + 4)^2 = 36$

According to the Question

$S-D=27 \Rightarrow 36-24=12 \neq 27 \therefore D \neq 24$

If D = 54 then $(5+4)^2-54=81-54=27$

therefore D is 54.

12. (a) The product of 44 and 11 is 484.

But given product of 44 and 11 = 3414 (in number system)

Here, $3x^3+4x^2+1x^1+4\times x^0=484$

$\Rightarrow 3x^3+4x^2+x=480$

This equation is satisfied only when x = 5.

In decimal system, the number 3111 can be written as 406 = $[3 \times 5^3 + 1 \times 5^2 + 1 \times 5^1 + 1 \times 5^0]$

13. (b) Every term in the question is either 1 or –1. In order to have zero the number of terms must be even. Note that there are n number of terms. (since the first term in each product varies from x_1 to x_n).

So n has to be even.

14. (b) 4-legged chairs = 4 legged tables = no. of workers.

3-legged stools = 4 legged almirahs

No. of stools = 1 + no. of workers.

Total no. of legs = 585

Let the no. of workers = x

$\therefore 2x + x \times 4 + x \times 4 + (x + 1)\,3 + (x + 1)4 = 585$

$\Rightarrow 2x + 4x + 4x + 7x + 7 = 585 \Rightarrow 17x = 578$

$\Rightarrow x = 34$

Hence, the no. of workers are 34.

15. (d) $B + D = 50$ ....(i)

$B + 8 = C \Rightarrow B = C - 8$ ....(ii)

$A - 8 = C - 3 \Rightarrow A = C + 5$ ....(iii)

$A + 6 = 2D \Rightarrow A + 6 = 2\,(50 - B)$

$\Rightarrow A + 2B = 94$ ....(iv)

Putting the values of (ii) and (iii) in (iv) we get

$C + 5 + 2\,(C - 8) = 94$

$\Rightarrow 3C - 11 = 94 \Rightarrow C = 105/3 = 35$

A = 40, B = 27, C = 35, D = 23.

16. (d) Suppose husband's age be H years.

Then wife's age W = H – 9

Son's age $S = \dfrac{H-9}{2}$

Daughter's age $D = \dfrac{H}{3}$

According to question,

$\dfrac{H}{3}+7=\dfrac{H-9}{2} \Rightarrow 2H+42=3H-27$

$\Rightarrow H=42+27=69$

$\therefore$ W = 60.

But conventional method like this will take time to solve the problem. Easier way is to solve through options. For example take the middle option. Selecting middle option is best strategy as it indicates towards possible answer. Moreover, a closer look reveals that 40+9 is not divisible by 3 and same is applicable for 50+9. Divisibility by 3 is desirable as husband's age is 3 times that of the daughter. This leaves options (c) and (d).

Solving through option (c).

45+9 = 54 H.

D = 18
S = 22.5 → Difference is 4.5 years, so this is incorrect.

Solving through option (d) matches all conditions.

17. (b) Legs – Heads = 69.

Cows = Hens, Cows = 2 Bullocks,

One keeper / ten birds and cattle.

Hence from the given data, we infer that the distribution is as follows :

Per keeper, there are 4 Hens, 4 Cows, 2 Bullocks.

In this case, (Heads $\approx$ Legs) = 23.

$\therefore$ There would be three times the above distribution in the farm.

$\therefore$ No. of cows = 12.

Alternative

The fact that there is one keeper per ten animals gives a hint that the number of heads is either 11 or multiple of 11. Taking eleven as base we get

Heads	B = 2	C = 4	H = 4	K = 1	Total = 11	Difference → 23.
Legs	B = 8	C = 16	H = 8	K = 2	Total = 34	

The difference in question is 69, which is 3 times of 23. Hence multiply everything by three, result is as follows:

Heads	B = 6	C = 12	H = 12	K = 3	Total = 33	Difference → 69.
Legs	B = 24	C = 48	H = 24	K = 6	Total = 102	

Hence (b) is the correct alternative.