CH 3 TRIGONOMETRY

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (a) π radians = $180°$

Therefore, $\frac{3\pi}{5}$ radians $= \frac{3 \times 180°}{5} = 108°$

2. (d) $\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A} = \frac{7\frac{\sin A}{\cos A} - 3}{7\frac{\sin A}{\cos A} + 2}$

(Dividing num. & denom. by cos A)

$= \frac{7 \times \frac{4}{7} - 3}{7 \times \frac{4}{7} + 2}$ $\left[\frac{\sin A}{\cos A} = \frac{4}{7}\right]$

$= \frac{4-3}{4+2} = \frac{1}{6}$

3. (c) For $0 < x \le \pi/2$

sin x and cosec x both are positive

We know that AM of two positive numbers $\ge$ their GM.

Thus, $\frac{\sin x + \operatorname{cosec} x}{2} \ge \sqrt{\sin x . \operatorname{cosec} x}$

or sin x + cosec x $\ge$ 2

Thus, the least value is 2.

4. (b) 7 cosec θ – 3 cot θ = 7

7 cosec θ = (7 + 3 cot θ)

or 49 $\operatorname{cosec}^2\theta$ = 49 + 9 $\cot^2\theta$ + 42 cotθ

or 49 (1 + $\cot^2\theta$) = 49 + 9 $\cot^2\theta$+ 42 cotθ

or 40 $\cot^2\theta$ = 42 cot θ or

cot $\theta = \frac{21}{20}$

Now, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{441}{400} = \frac{841}{400}$

or coseθ = $\frac{29}{20}$

Therefore, 7 cotθ – 3cosec θ $= 7 \times \frac{21}{20} - 3 \times \frac{29}{20}$

$= \frac{147}{20} - \frac{87}{20} = \frac{60}{20} = 3$

5. (b) sin θ + cosθ $= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right)$

$= \sqrt{2}\left(\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right) = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

Max. value of $\sin\left(\theta + \frac{\pi}{4}\right) = 1$

when $\theta + \frac{\pi}{4} = \frac{\pi}{2}$ or $\theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

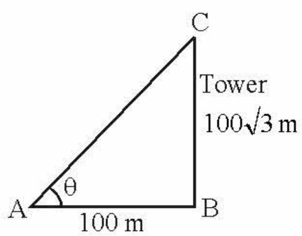
Hence, $\sin\theta + \cos\theta$ is max. at $\theta = \pi/4$

6. (d) $\cos^6 x + \sin^6 x + 3\sin^2 x \cos^2 x$

$= (\cos^2 x)^3 + (\sin 2x)^3 + 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$

$= (\cos^2 x + \sin^2 x)^3 = 1$

7. (c)

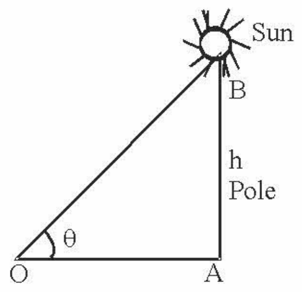


θ is the angle of elevation of the top of the tower (C).

$\therefore \quad \tan\theta = \frac{100\sqrt{3}}{100} = \sqrt{3}$

$\Rightarrow \quad \theta = 60$

8. (b) In rt. ΔOAB, $\frac{AB}{OA} = \tan\theta$



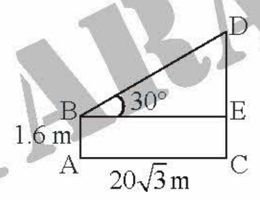
$\frac{h}{h} = \tan\theta$

$(\because OA = AB = h)$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$.

9. (a) Let AB be the observer and CD be the tower.

Draw $BE \perp CD$.

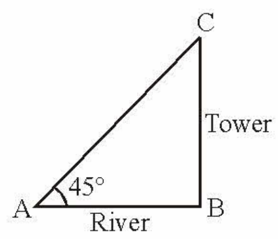


Then, CE = AB = 1.6 m, BE = AC = $20\sqrt{3}$ m.

$\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}} \quad \Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}} m = 20 m.$

$\therefore$ CD = CE + DE = (1.6 + 20)m = 21.6 m.

10. (c)

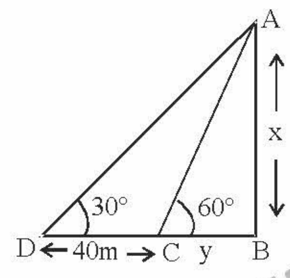


AB is the river and BC is the tower.

$\therefore \frac{BC}{AB} = \tan 45° = 1 \Rightarrow BC = AB$

11. (d) Let height of tower AB = x m and BC = ym, DC = 40m.

In ΔABC,



$\frac{AB}{BC} = \tan 60° \quad \Rightarrow \frac{x}{y} = \sqrt{3} \Rightarrow y = \frac{x}{\sqrt{3}}$ ....(i)

Now In rt Δ ABD, $\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{x}{40 + y} = \frac{1}{\sqrt{3}}$

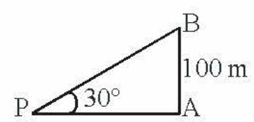
$\Rightarrow \sqrt{3}x = 40 + y \quad \Rightarrow \sqrt{3}x = 40 + \frac{x}{\sqrt{3}}$ [using (i)]

$\Rightarrow 3x = 40\sqrt{3} + x \quad \Rightarrow 3x - x = 40\sqrt{3}$

$\Rightarrow 2x = 40\sqrt{3}$

$x = 20\sqrt{3}$m ; $\therefore y = \frac{20\sqrt{3}}{\sqrt{3}}$ = 20 m.

12. (c) Let B be the tower. Then, $\angle APB$ = 30° and AB = 100 m.

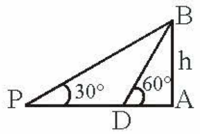


$\frac{AB}{AP} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow (AB \times \sqrt{3}) = 100\sqrt{3}$m.

$= (100 \times 1.73)$ m

$= 173$ m.

13. (d)



One of AB, AD and CD must have been given.

So, the data is inadequate.

14. (a) Let h be the height.

Then, 6 : 4 = h : 50

$\Rightarrow \ h = \dfrac{50 \times 6}{4} = 75\,\text{m}$

15. (c) The broken part will become the hypotenuse and the remaining part will be perpendicular so angle opposite to it become 30°. Since side opposite to 30° angle is half of the hypotenuse.

16. (2) $\sin\alpha + \cos\beta = 2$

$\sin\alpha \le 1; \cos\beta \le 1$

$\Rightarrow \ \alpha = 90°; \beta = 0°$

$\therefore \ \sin\left(\dfrac{2\alpha+\beta}{3}\right) = \sin\left(\dfrac{180°}{3}\right)$

$= \sin 60° = \dfrac{\sqrt{3}}{2}$

17. (3) $\cos^4\theta - \sin^4\theta = \dfrac{2}{3}$

$\Rightarrow \ (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= \dfrac{2}{3}$

$\Rightarrow \ \cos^2\theta - \sin^2\theta = \dfrac{2}{3}$

$\Rightarrow \ \cos^2\theta - (1 - \cos^2\theta) = \dfrac{2}{3}$

$\Rightarrow \ 2\cos^2\theta - 1 = \dfrac{2}{3}$

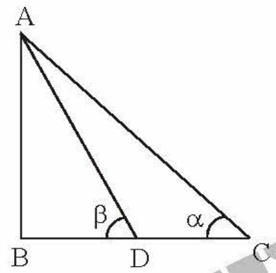
18. (3) cot 10°. cot 80°. cot 20°. cot 70°. cot 60°

= cot 10°. tan 10°. cot 20°. tan 20°. cot 60°

$\left[\begin{array}{r} \because \tan(90° - \theta) = \cot\theta \\ \tan\theta.\cot\theta = 1 \end{array}\right]$

$= 1\ .1.\sqrt{3} = \sqrt{3}$

19. (3)



AB = monument = h metre

DC = 138 metre

BD = × metre

$\tan\alpha = \dfrac{1}{5}$

$\sec\beta = \dfrac{\sqrt{193}}{12}$

$\therefore \ \tan\beta = \sqrt{\sec^2\beta - 1}$

$= \sqrt{\dfrac{193}{144} - 1} = \sqrt{\dfrac{193 - 144}{144}}$

$= \sqrt{\dfrac{49}{144}} = \dfrac{7}{12}$

$\therefore$ From ΔABC,

$\tan\alpha = \dfrac{AB}{BC} \Rightarrow \dfrac{1}{5} = \dfrac{h}{x + 138}$

$\Rightarrow \ h = \dfrac{x + 138}{5}$

$\Rightarrow \ 5h = x + 138$ ....(i)

From ΔABD,

$\tan\beta \frac{h}{x} \Rightarrow \frac{7}{12} = \frac{h}{x}$

$\Rightarrow \quad x = \frac{12h}{7}$ .....(ii)

$\therefore \quad 5h = \frac{12h}{7} = 138$

$\Rightarrow \quad 35h - 12h = 138 \times 7$

$\Rightarrow \quad 23h = 138 \times 7$

20. (1) $\frac{\sin\alpha}{\cos(30^\circ + \alpha)} = 1$

$\Rightarrow \quad \frac{\sin\alpha}{\sin(90^\circ - 30 - \alpha)} = 1$

$\Rightarrow \quad \frac{\sin\alpha}{\sin(60^\circ - \alpha)} = 1$

$\Rightarrow \quad \sin\alpha = \sin(160^\circ - \alpha)$

$\Rightarrow \quad \alpha = 60^\circ - \alpha$

$\Rightarrow \quad 2\alpha = 60^\circ \Rightarrow \alpha = 30^\circ$

$\therefore \quad \sin\alpha + \cos 2\alpha$

$= \sin 30^\circ + \cos 60^\circ$

$= \frac{1}{2} + \frac{1}{2} = 1$

21. (1) $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

$\therefore \quad \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$

$$= \frac{8 \times \frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$$

$$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$$

22. (1) $\cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \quad \cos^4\theta = 1 - \cos^2\theta = \sin^2\theta$

$\Rightarrow \quad \tan^2\theta = \cos^2\theta$

$\therefore \quad \tan^2\theta + \tan^4\theta = \cos^2\theta + \cos^4\theta = 1$

23. (2) $\tan 4^\circ . \tan 43^\circ . \tan 47^\circ . \tan 86^\circ$

$= (\tan 4^\circ . \tan 86^\circ)(\tan 43^\circ . \tan 47^\circ)$

$= (\tan 4^\circ . \cot 4^\circ) . (\tan 43^\circ . \cot 43^\circ)$

$= 1$

$$\left[\begin{array}{r} \tan(90^\circ - \theta) = \cot\theta; \\ \tan\theta . \cot\theta = 1 \end{array}\right]$$

24. (3) $\tan\theta = \frac{4}{3}$ (Given)

$\therefore \quad \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3\tan\theta + 2}{3\tan\theta - 2}$

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4 + 2}{4 - 2} = 3$$

25. (4) $\sec 17^\circ - \sin 73^\circ$

$= \sec 17^\circ - \sin(90^\circ - 17^\circ)$

$= \sec 17^\circ - \cos 17^\circ$

$= \frac{1}{\cos 17^\circ} - \cos 17^\circ$

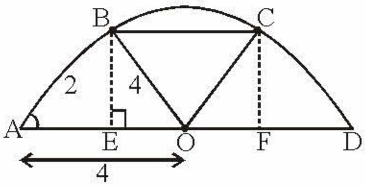
$= \frac{1 - \cos^2 17^\circ}{\cos 17^\circ} = \frac{\sin^2 17}{\cos 17^\circ}$

$$= \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}}$$

$$= \frac{\frac{x^2}{y^2}}{\frac{\sqrt{y^2-x^2}}{y}} = \frac{x^2}{y\sqrt{y^2-x^2}}$$

EXERCISE 2

1. (b)

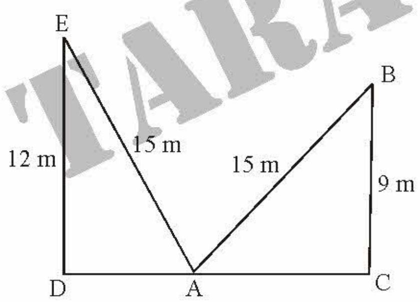


BO = radius = 4 = AO

$$AE = 2\cos A = 2\times\left(\frac{2^2+4^2-4^2}{2\times2\times4}\right) = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ BC = AD – AE – FD $= 8-\frac{1}{2}-\frac{1}{2} = 7$ ($\because$ AE = FD)

2. (b)



AB = AE is the ladder = 15 m

B is the window. A is the foot of the ladder.

CD is the width of the street.

Window is at a height of 12 m.

$\Rightarrow$ BC = 9 m. Also ED = 12 m

$\therefore$ Width of the street = AD + AC ...(1)

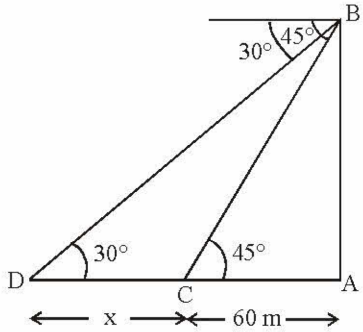
Now from triangle BAC and EAD, we have

$AC^2 + 9^2 = 15^2$ and $AD^2 + 12^2 = 15^2$

$\Rightarrow$ AC = 12 and AD = 9

From (1) we get, Width of the street = 21 metres.

3. (a)



From Δs ΔBCA and ΔBDA

$$\frac{BA}{60} = \tan 45° \text{ and } \frac{BA}{60+x} = \tan 30°$$

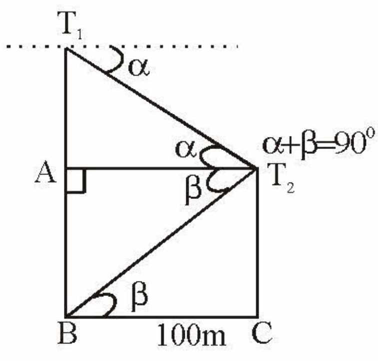
$$\Rightarrow 60 = (60+x) \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow 60+x = \sqrt{3}\times 60 \Rightarrow x = 60(\sqrt{3}-1)$$

$\therefore$ The distance of $60(\sqrt{3}-1)$ meters is covered by the boat in 5 seconds.

$\therefore$ Speed of the boat per hour(in km).

4. (c) Let α and β be the angles of depression of the men on T_1 and T_2 respectively



Now, $\alpha + \beta = 90^0$.

$\alpha = \beta = 45^0$. as the

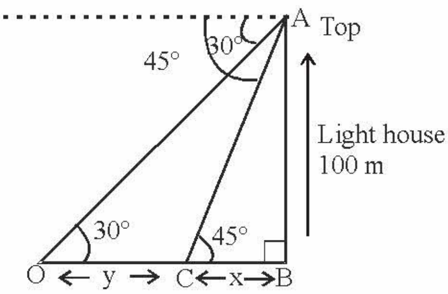
ΔT_1AT_2 and ΔBAT_2 are congruent right angled triangles.

$\therefore \Delta ABT_2$ is an isosceles right angled triangle

Hence $T_1B = 2T_2C = 2BC = 200$ m

5. (b) Let 'y' be the required distance between two positions O and C of the ship

In rt. ΔABC,



$\cot 45° = \frac{x}{100} \Rightarrow x = 100$ ...(i)

In ΔAOB, $\frac{y+x}{100} = \cot 30°$

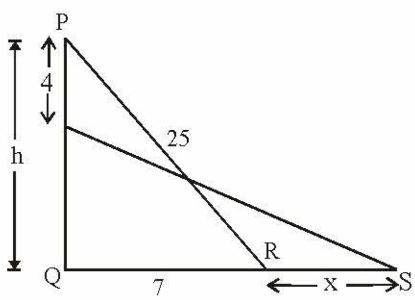
$\Rightarrow y + x = 100\sqrt{3} \Rightarrow y = 100\sqrt{3} - x$

$\Rightarrow y = 100\sqrt{3} - 100$ [using (i)]

$\Rightarrow y = 100(\sqrt{3} - 1)$

$\Rightarrow y = 100(1.732 - 1) = 100 \times 0.732 =$ 73.20 m.

6. (b) Let the height of the wall be h.



Now, $h = \sqrt{25^2 - 7^2}$

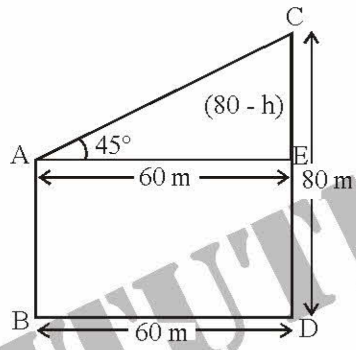
$= \sqrt{576} = 24$m

QS $= \sqrt{625 - 400}$

$= \sqrt{225} = 15$m

Required distance, x = (15 – 7) = 8 m

7. (a) Let AB and CD are the two trees.



Given: CD = 80 m

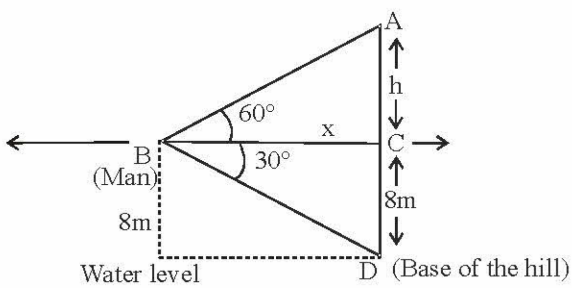
From ΔAEC, $\frac{80 - AB}{60} = \tan 45°$

$\Rightarrow$ 80 – AB = 60 $\Rightarrow$ AB = 20

$\therefore$ Height of the first tree = 20.

8. (c) Let x be distance of the hill from man and (h + 8) m be the height of the hill

Now, in rt Δ ACB,



$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

......(i)

In rt Δ BCD, tan 30° $= \frac{8}{x} \Rightarrow x = 8\sqrt{3}$

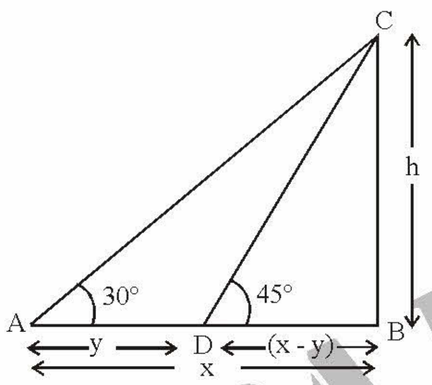
......(ii)

$\therefore$ Height of the hill = h + 8 $= \sqrt{3} \times 8\sqrt{3} + 8$

[using (i) & (ii)]

$= 24 + 8 = 32$ m

9. (b) Let h be the height of the tower. It takes 12 minutes to reach the car at the point D from the point A, i.e. the distance of y meters is covered by the car in 12 minutes.



$\therefore \frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{x}{\sqrt{3}}$ ...(i)

$\frac{h}{x-y} = \tan 45° = 1$

$\Rightarrow h = x - y$ ...(ii)

(i) and (ii)

$\Rightarrow \frac{x}{\sqrt{3}} = x - y$

$\Rightarrow y = x - \frac{x}{\sqrt{3}} = x\left[\frac{\sqrt{3}-1}{\sqrt{3}}\right]$

$\Rightarrow x - y = \frac{x}{\sqrt{3}}$

This to cover distance AD = y

$= x\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$ units, the car takes 12 minutes

$\therefore$ To cover the distance DC = x – y

$= x - \frac{x(\sqrt{3}-1)}{\sqrt{3}} = \frac{x[\sqrt{3}-\sqrt{3}+1]}{\sqrt{3}} = \frac{x}{\sqrt{3}}$,

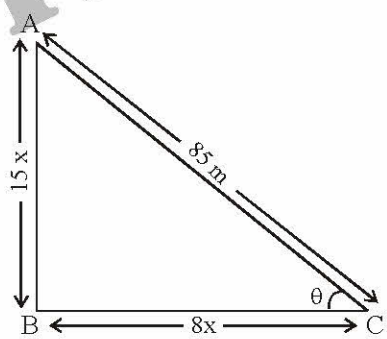
the car will take

$\frac{12\times\sqrt{3}}{x(\sqrt{3}-1)} \times \frac{x}{\sqrt{3}}$

$= \frac{12}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{12\sqrt{3}+12}{3-1} = 6\sqrt{3}+6$

= 16.392 minutes $\approx$ 16 min. 23 seconds

10. (b)



We have, $\tan\theta = \frac{15}{8}$

Let AB = 15x and BC = 8x

Now $(AC)^2 = (AB)^2 + (BC)^2$

or $(85)^2 = (15x)^2 + (8x)^2$

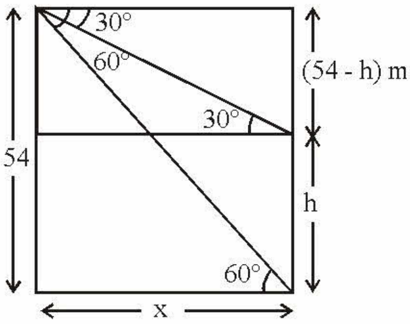
or $(85)^2 = 225x^2 + 64x^2$

or $x^2 = 25$

or $x = 5$

Height of the kite = 15 × 5 = 75 m.

11. (b)



Let width of the river be x metres and height of the other temple be h meters.

$\therefore \quad \tan 30° = \frac{54-h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{54-h}{x}$

$\Rightarrow x = \sqrt{3}(54-h)$
...(1)

Now , $\tan 60° = \frac{54}{x}$

$\Rightarrow \quad x = \frac{54}{\sqrt{3}} = 18\sqrt{3}$ ...(2)

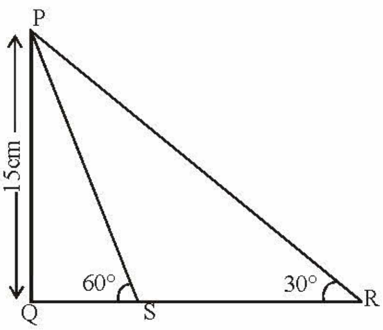
From (1) and (2), we get = 18 × 1.73 = 31.14m

$\sqrt{3}(54-h) = 18\sqrt{3} \Rightarrow h = 36$

$\therefore$ Width of the river = 31.14m

and height of the other temple = 36m

12. (c)



We have, $\tan 60° = \frac{15}{QS}$

$\Rightarrow QS = \frac{15}{\sqrt{3}} = 5\sqrt{3}$ ...(i)

Again, $\tan 30° = \frac{15}{QR}$

$\Rightarrow QR = 15\sqrt{3}$ ...(ii)

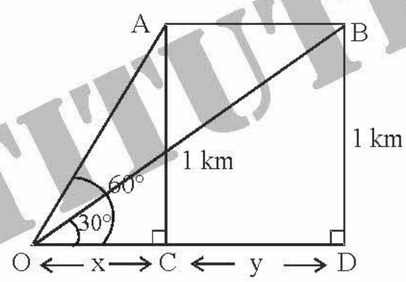
Therefore, RS = QR – QS

$= 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3}$

or, $RS = 10\sqrt{3}m$

13. (d) Let 'O' be the observation point, and let A be the first position of aeroplane such that $\angle AOC = 60°$.

AC = 1km = BD



Let B be the position of aeroplane after 10 seconds, $\angle BOD = 30°$, OC = x km and CD = y km.

Now, in rt ΔOCA,

$\cot 60° = \frac{x}{1}$

$\Rightarrow x = \frac{1}{\sqrt{3}}$ ....(i)

In ΔODB, $\cot 30° = \frac{x+y}{1}$

$\Rightarrow x + y = \sqrt{3}$ ....(ii)

Subtract (i) – (ii)

$-y = \frac{1}{\sqrt{3}} - \sqrt{3}$

$\Rightarrow -y = \frac{1-3}{\sqrt{3}} \Rightarrow y = \frac{2}{\sqrt{3}}$ km

In 10 seconds, distance covered $y = \frac{2}{\sqrt{3}}$ km

$\therefore$ Speed of the plane

$$= \frac{2}{\sqrt{3}} \times \frac{1}{10} \times 60 \times 60 \text{ km/hr}$$

$$= 240\sqrt{3} \text{ km/hr}$$

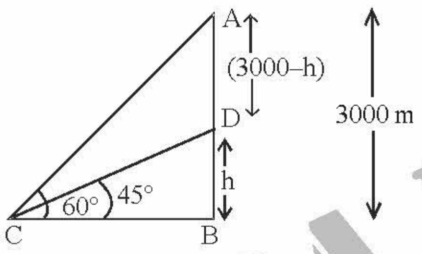
14. (c) Let A and D be the positions of two aeroplanes

AD = AB – DB = 3000 – h

Now, In Δ DBC

$$\tan 45° = \frac{h}{BC}$$

$$\Rightarrow 1 = \frac{h}{BC} \Rightarrow BC = h$$



Again, $\tan 60° = \frac{3000}{BC} = \frac{3000}{h}$

or $h = \frac{3000}{\sqrt{3}}$

$$\Rightarrow AD = 3000 - h = 3000\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$= 3000\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \text{(Rationalizing)}$$

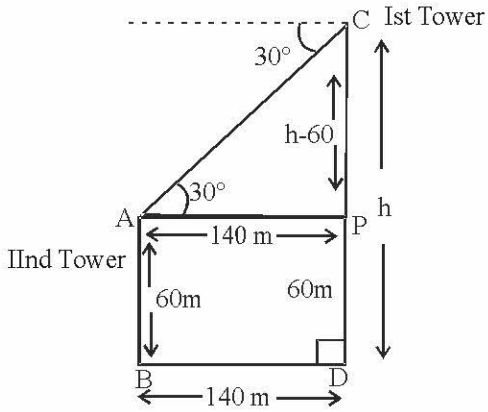
or $\quad AD = \frac{3000}{3}\left(3 - \sqrt{3}\right)$

= 1000 (3 – 1.732)

= 1000(1.268) = 1268 m

15. (b) Let height of Ist tower CD = h m

and height of IInd tower AB = 60 m



$\therefore$ CP = h - 60

Now, in rt ΔAPC

$$\tan 30° = \frac{CP}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CP}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-60}{140}$$

$$\Rightarrow (h-60)\sqrt{3} = 140$$

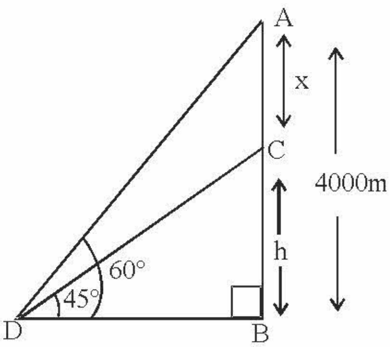
$$\Rightarrow \sqrt{3}h - 60\sqrt{3} = 140 \Rightarrow \sqrt{3}h = 140 + 60\sqrt{3}$$

$$\Rightarrow h = \frac{140 + 60\sqrt{3}}{\sqrt{3}} = 140.73\text{m}$$

16. (a) Let A and C be the positions of two planes.

Let BC = h m.

Now, in Δ DCB, $\cot 45° = \frac{DB}{BC} \Rightarrow \frac{BD}{h} \Rightarrow BD = h$



Now, in rt Δ ADB, $\cot 60^\circ = \dfrac{BD}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{BD}{4000} \Rightarrow BD = \frac{4000}{\sqrt{3}}$$

$$\Rightarrow h = \frac{4000}{\sqrt{3}} \quad (\because BD = BC)$$

Now, AC = AB - BC

$$= 4000 - \frac{4000}{\sqrt{3}} = 4000\left[1 - \frac{1}{\sqrt{3}}\right]$$

$$= 4000\left[\frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right] \quad \text{(Rationalising)}$$

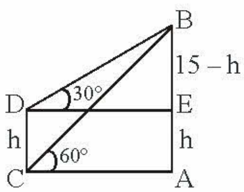
$$= 4000\left[\frac{3-\sqrt{3}}{3}\right] = \frac{4000}{3}\left[3-\sqrt{3}\right]$$

$$= \frac{4000}{3}\left[3 - 1.732\right]$$

$$= \frac{4000}{3} \times 1.27 = 1693.3\text{m}$$

17. (c) Let AB be the tower and CD be the electric pole. Then,

$\angle ACB = 60^\circ$, $\angle EDB = 30^\circ$ and $AB = 15$ m.



Let CD = h.

Then, BE = (AB – AE)

= (AB – CD) = (15 – h).

Now, $\tan 60^\circ = \dfrac{AB}{AC} = \sqrt{3} \Rightarrow AC = \dfrac{AB}{\sqrt{3}} = \dfrac{15}{\sqrt{3}}$.

And, $\dfrac{BE}{DE} = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow DE = BE\sqrt{3} = \sqrt{3}(15 - h).$$

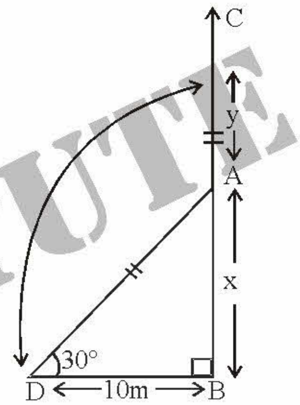
Now, AC = DE

$$\Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3}(15 - h)$$

$$\Rightarrow 3h = (45 - 15) \Rightarrow h = 10\text{m}.$$

18. (b) Let CAB be the tree which breaks at A due to storm . Let AB = x m and AC = y m $\Rightarrow$ AD = y m

Now, In rt Δ ABD,



$$\tan 30^\circ = \frac{x}{10}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$\Rightarrow x = \frac{10}{\sqrt{3}} \quad \ldots(i)$$

and $\sec 30^\circ = \dfrac{y}{10}$

$$\Rightarrow \frac{2}{\sqrt{3}} = \frac{y}{10} \Rightarrow y = \frac{20}{\sqrt{3}} \quad \ldots(ii)$$

Add (i) and (ii) i.e. height of tree

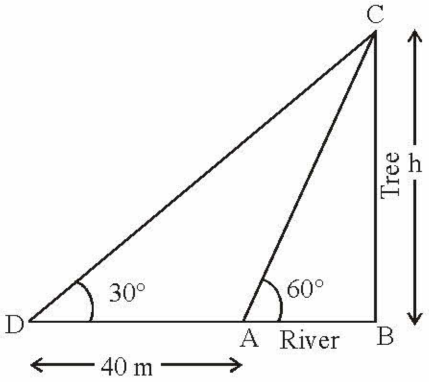
$$BC = x + y = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

[By Rationalising]

$$= 10\sqrt{3}\text{m}.$$

19. (b)



$$\frac{BC}{AB} = \tan 60° = \sqrt{3} \Rightarrow BC = \sqrt{3}AB$$

$$\frac{BC}{AB+40} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BC = \frac{AB+40}{\sqrt{3}}$$

$$\therefore \quad \sqrt{3}AB = \frac{AB+40}{\sqrt{3}}$$

Then, AB = 20

$\therefore$ Breadth of river = 20 m

20. (b) Figure shows the tower OP standing at O, the midpoint of BC. We are given $\angle PAO = 45° = \angle PBO = \angle PCO$.

Let the height of the tower be h meter. Clearly, $\Delta PAO \cong \Delta PBO \cong \Delta PCO$ (AAS direction)

In Δ PAO,

$$\frac{OP}{OA} = \tan 45° \Rightarrow OA = h\cot 45° = h \qquad ...(1)$$

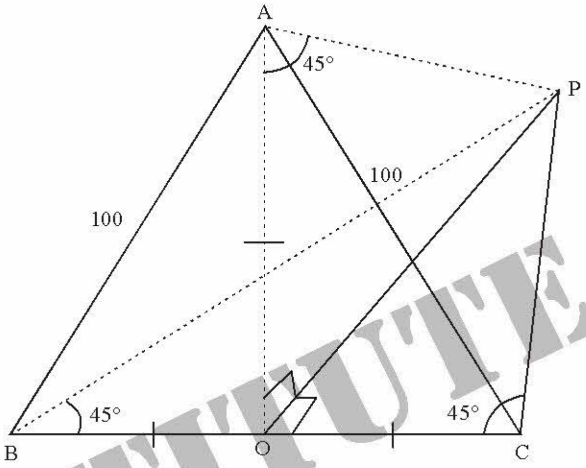
In Δ BOP,

$$\frac{OP}{OB} = \tan 45° \Rightarrow OB = OP \Rightarrow OB = h \qquad ... (2)$$

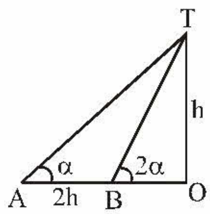
In Δ AOB, $AO^2 + OB^2 = AB^2$

$\Rightarrow$ $h^2 + h^2 = 100^2$ [From (1) and (2)]

$\Rightarrow$ $h = 50\sqrt{2}$ m



21. (c). In the following figure,



A is the point where the professor was standing initially.

OT = h = highest of the tower.

B is the point where the professor is standing after walking the distance 2h.

h cot α = OA, h cot 2α = OB

Therefore, AB = OA – OB = 2h

= h cot α – h cot 2α.

$\Rightarrow 2 = \cot\alpha - \cot 2\alpha$

$$\Rightarrow \quad 2 = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos 2\alpha}{\sin 2\alpha}$$

$\Rightarrow$ cos α $\times$ sin 2α – sin α $\times$ cos 2α = 2 sin α $\times$ sin 2α

$\Rightarrow$ sin $(2\alpha - \alpha)$ = 2 sin α $\times$ sin 2α

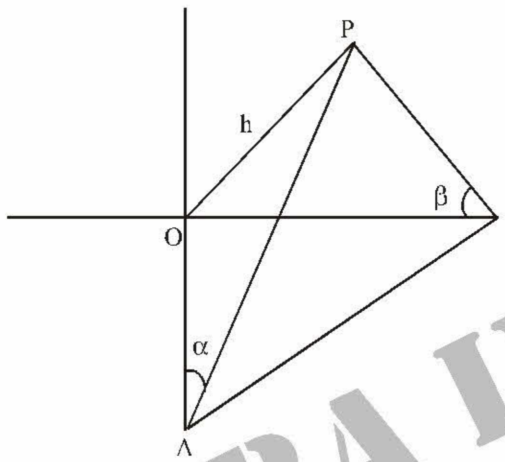
$\Rightarrow$ 1 = 2 sin 2α

$\Rightarrow$ sin 2α = 1/2

$\Rightarrow$ 2α = 30° or α = 15°.

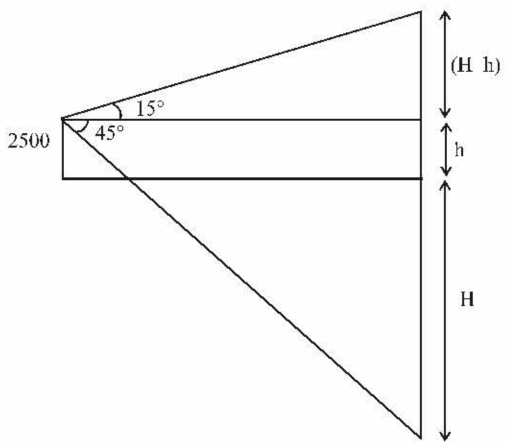
22. (c). $OB = h\cot b,\ OA = h\cot a$

$$h^2 = \frac{d^2}{\cot^2 b + \cot^2 a} \text{ Þ } h = \frac{d}{\sqrt{\cot^2 b + \cot^2 a}}$$



23. (a). (H – h) cot 15° = (H + h) cot 45°

or $H = \frac{h(\cot 15° + 1)}{(\cot 15° - 1)}$



Since, h = 2500 and substitute

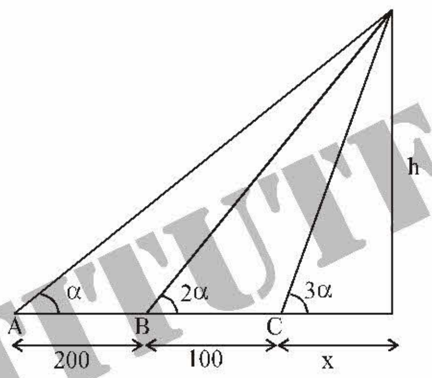
cot 15° = $2+\sqrt{3}$, we get, $H = 2500\sqrt{3}$

24. (d).

$x = h\cot 3a$ ... (1)

$(x+100) = h\cot 2a$ ... (2)

$(x+300) = h\cot a$ ... (3)



From (1) and (2), $-100 = h(\cot 3a - \cot 2a)$

From (2) and (3), $-200 = h(\cot 2a - \cot a)$

$$100 = h\left(\frac{\sin\alpha}{\sin 3\alpha \sin 2\alpha}\right) \text{ and}$$

$$200 = h\left(\frac{\sin\alpha}{\sin 2\alpha \sin\alpha}\right)$$

or $\frac{\sin 3\alpha}{\sin\alpha} = \frac{200}{100} \Rightarrow \frac{\sin 3\alpha}{\sin\alpha} = 2$

$3\sin a - 4\sin^3 a - 2\sin a = 0$

$4\sin^3 a - \sin a = 0 \text{ Þ } \sin a = 0$

or $\sin^2\alpha = \frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right) \Rightarrow \alpha = \frac{\pi}{6}$

Hence, $h = 200\sin\frac{p}{3} = 200\frac{\sqrt{3}}{2} = 100\sqrt{3}$

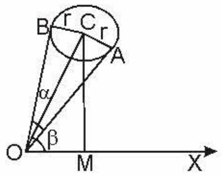
{from (2)}

25. (b) Let C be the centre of the balloon and O be the position of the observer at the horizontal line OX. Let OA and OB be the tangents to the balloon. It is given that $\angle$ AOB = α and $\angle$ XOC = β.

Let r be the radius of the ballon,

then, CA = CB = r

Clearly, the right angled triangle



OAC and OBC are congruent.

Let r be the right angled

triangles OAC and OBC are congruent

$\therefore \angle$ AOC = $\angle$ BOC = $\alpha/2$

In Δ OAC, $\sin\frac{a}{2} = \frac{r}{OC}$

$\Rightarrow$ OC = r $\operatorname{cosec}\frac{a}{2}$ ....(1)

Let CM be perpendicular from C on OX.

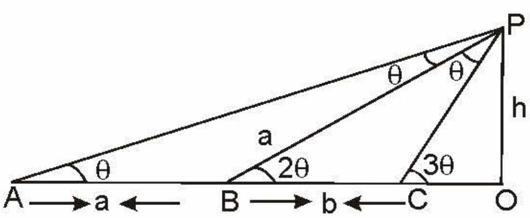
In Δ OCM, $\sin\beta = \frac{CM}{OC}$

$\Rightarrow$ CM = OC $\sin\beta$ = α $\operatorname{cosec}\frac{r}{2}\sin\beta$ [By (1)]

Hence, the height of the centre of the balloon is

$r \operatorname{cosec}\frac{a}{2}\sin\beta$

26. (b). Let the object be at P at a height h form OA. Let the object when observed from A, B and C the angles of elevation are θ , 2θ and 3θ respectively.



From Δ PAB, $2\theta = \theta + \angle APB$

$\Rightarrow \angle APB = \theta$

$\therefore \angle PAB = \angle APB = \theta$

$\Rightarrow$ AB = BP = a

Similarly, in triangle BPC, $\angle BPC = \theta$

From Δ OBP, $\sin 2\theta = h/a$

$\Rightarrow h = a\sin 2\theta$

$\Rightarrow$ $h = 2a\sin\theta\cos\theta$ ...(1)

From Δ PBC, $\frac{PB}{\sin(180^\circ - 3\theta)} = \frac{BC}{\sin\theta}$ [By sine rule]

$\Rightarrow \frac{a}{\sin 3\theta} = \frac{b}{\sin\theta}$

$\Rightarrow \frac{a}{b} = \frac{\sin 3\theta}{\sin\theta}$

$\Rightarrow \frac{a}{b} = \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}$

$\Rightarrow \frac{a}{b} = 3 - 4\sin^2\theta$

$\Rightarrow 4\sin^2\theta = 3 - \frac{a}{b}$

$\Rightarrow \sin^2\theta = \frac{3b-a}{4b}$

$\Rightarrow \sin\theta = \sqrt{\frac{3b-a}{4b}}$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

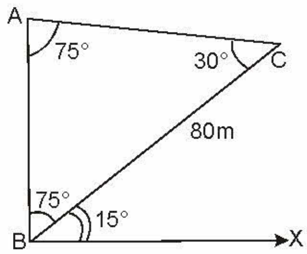
$\Rightarrow \cos^2\theta = 1 - \frac{3b-a}{4b} = \frac{a+b}{4b}$

$\Rightarrow \cos\theta = \sqrt{\frac{a+b}{4b}}$

Putting the values of $\sin\theta$ and $\cos\theta$ in (1), we get

$h = 2a\sqrt{\frac{3b-a}{4b}}.\sqrt{\frac{a+b}{4b}} = \frac{a}{2b}\sqrt{(a+b)(3b-a)}$

27. **(b).** Let BC be the declivity and BA be the tower



By sine formula $\frac{BC}{\sin 75°} = \frac{AB}{\sin 30°}$

$$\Rightarrow AB = \frac{80 \sin 30°}{\sin 75°} = \frac{40}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{80\sqrt{2}}{\sqrt{3}+1}$$

$$= \frac{80\sqrt{2}}{\sqrt{3}+1} . \frac{\sqrt{3}-1}{\sqrt{3}-1} = 40\sqrt{2}(\sqrt{3}-1) = 40(\sqrt{6}-\sqrt{2})$$