Answers and Explanations

1. (b) $\sec x = \operatorname{cosec} y$

$\Rightarrow \cos x = \sin y$

$\Rightarrow \sin\left(\frac{\pi}{2} - x\right) = \sin y$

$\Rightarrow y = \frac{\pi}{2} - x$

$\Rightarrow x + y = \frac{\pi}{2}$

$\therefore \sin(x + y) = \sin\frac{\pi}{2} = 1$

2. (a) tan1°. tan2°. tan3°........ tan45°......tan88° tan 89°

= tan (1°. tan89°) (tan 2°. tan 88°)..........tan 45°

= (tan 1°. cot1°). (tan2°. cot2°)..........

[∵ tan (90° – θ) = cotθ, tanθ, cotθ = 1]

3. (c) $A + B + C = \pi$

$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \sin\left(\frac{A+B}{2}\right)$

$= \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos\frac{C}{2}$

Similarly,

$\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

$\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

$\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$

4. (c)

$(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$

$= \sec^2 A + \cos^2 A - 2\sec A \cos A + \operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A. \sin A - co^2 A - \tan^2 A + 2 \cot A. \tan A$

$= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$

$= 3 - 1 = 1$

$\left[\because \sec A. \cos A = 1; \sin A. \operatorname{cosec} A = 1; \tan A. \cot A = 1 \text{ etc}\right]$

5. (b) $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\frac{\sin^2\theta}{\cos^2\theta} + 3 = \frac{4}{\cos^2\theta} = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 7\tan^2\theta - 4\tan^2\theta = 4 - 3$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

6. (a) No. of terms in 1 + 5 + 9 + ...+ 89 = n

$\therefore a + (n - 1)d = t_n$

$\Rightarrow 1 + (n - 1)4 = 89$

$\Rightarrow (n - 1)4 = 89 - 1 = 88$

$\Rightarrow n - 1 = 22$

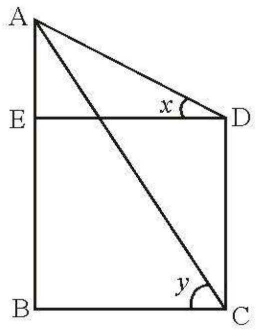
$\Rightarrow n = 23$

$\Rightarrow$ Now, $\sin^2 1° + \sin^2 89° + \sin^2 5° + \sin^2 85° + ...+$ to 22 terms $+ \sin^2 45°$

$= (\sin^2 1° + \cos^2 1°) + (\sin^2 5° + \cos^2 5°) + ...+$ to 11 terms $+ \left(\frac{1}{\sqrt{2}}\right)^2$

$= 11 + \frac{1}{2} = 11\frac{1}{2}$ $\left[\begin{array}{l}\sin(90° - \theta) = \cos\theta \\ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$

7. (c)



CD = tree = h metre

AB = building = a metre

BC = ED = b metre

$\therefore$ From ΔAED,

$$\tan x = \frac{AE}{ED} \Rightarrow \tan x = \frac{a-h}{b}$$

$\Rightarrow$ $b = (a-h)\cot x$ .....(i)

From ΔABC,

$$\tan y = \frac{AB}{BC}$$

$$\Rightarrow \tan y = \frac{a}{b}$$

$\Rightarrow$ $b = a\cot y$ .....(ii)

From equations (i) and (ii),

$(a-h)\cot x = a\cot y$

$\Rightarrow$ $a\cot x - h\cot x = a\cot y$

$\Rightarrow$ $h\cot x = a(\cot x - \cot y)$

$$\Rightarrow a = \frac{h\cot x}{\cot x - \cot y}$$

8. (a) cot 18°

$$\left(\cot 72^\circ.\cos^2 22^\circ + \frac{1}{\tan 72^\circ.\sec^2 68^\circ}\right)$$

$$= \cot 18^\circ.\cot 72^\circ.\cos^2 22^\circ + \frac{\cot 18^\circ}{\tan 72^\circ.\sec^2 68^\circ}$$

$$= \cot 18^\circ.\tan 18^\circ.\cos^2 22^\circ + \frac{\cot 18^\circ}{\cot 18^\circ}.\cos^2 68^\circ$$

$$= \cos^2 22^\circ + \cos^2 68^\circ$$

$$= \cos^2 22^\circ + \sin^2 22^\circ = 1$$

$$\left[\begin{array}{r}\because \tan(90^\circ-\theta) = \cot\theta;\\ \sin(90^\circ-\theta) = \cos\theta;\\ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$$

9. (b) $2\sin^2\theta + 3\cos^2\theta$

$$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$$

$$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$$

$$= 2 + \cos^2\theta$$

$\because$ Minimum value of $\cos\theta = -1$

$\therefore$ Required minimum value = 2 + 1 + 3

10. (c)

$$\frac{1}{\text{cosec}^2 51^\circ} + \sin^2 39^\circ + \tan^2 51^\circ$$

$$- \frac{1}{\sin^2 51^\circ.\sec^2 39^\circ}$$

$$= \sin^2 51^\circ + \sin^2 39^\circ + \tan^2(90^\circ - 39^\circ)$$

$$- \frac{1}{\sin^2(90^\circ-39^\circ).\sec^2 39^\circ}$$

$$= \cos^2 39^\circ + \sin^2 39^\circ + \cot^2 39^\circ$$

$$- \frac{1}{\cos^2 39^\circ.\sec^2 39^\circ}$$

$[\because \sin(90^\circ-\theta) = \cos\theta), \tan(90^\circ-\theta) = \cot\theta]$

$$= 1 = \cot^2 39^\circ - 1$$

$$= \text{cosec}^2 39^\circ - 1 = x^2 - 1$$

11. (c) tan 4°. tan 43°. tan 47°. tan 86°

= tan 4°. tan 43°. tan (90° – 43°). tan (90° – 4°)

= tan 4°, tan 43°, cot 43°. cot 4° = 1

[tan (90° – θ) = cot θ; tan θ. cot θ = 1]

12. (b) $\dfrac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = \dfrac{2}{1}$

By componendo and dividendo,

$\dfrac{2\tan\theta}{2\cot\theta} = \dfrac{3}{1}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta} = 3$

$\Rightarrow \sin^2\theta = 3\cos^2\theta$

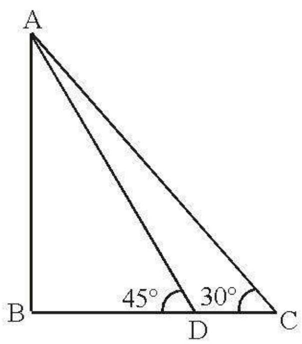
$\Rightarrow \sin^2\theta = 3\,(1-\sin^2\theta)$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin^2\theta = \dfrac{3}{4}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$

13. (d)



Let AB be a pillar of height h metre.

If BC = length of shadow = x, then

BD = $(x + 20)$ metre

From Δ ABC,

$\tan 45° = \dfrac{h}{x} \Rightarrow h = x$ .....(i)

From Δ ABD,

$\tan 30° = \dfrac{AB}{AD} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+20}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{h+20} \Rightarrow \sqrt{3}h = h + 20$

$\Rightarrow \left(\sqrt{3}-1\right)h = 20 \Rightarrow h = \dfrac{20}{\sqrt{3}-1}$

$= \dfrac{20}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{20\left(\sqrt{3}+1\right)}{2} = 10\left(\sqrt{3}+1\right)$ metre

14. (b) When $\theta = 0°$

$\sin^2\theta + \cos^4\theta = 1$

When $\theta = 45°$,

$\sin^2\theta + \cos^4\theta = \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$

When $\theta = 30°$

$\sin^2\theta + \cos^4\theta = \dfrac{1}{4} + \dfrac{9}{16} = \dfrac{13}{16}$

15. (d) $\sin\theta + \text{cosec}\theta = 2$

$\Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\therefore \sin^5\theta + \text{cosec}^5\theta = 1 + 1 = 2$

16. (d) $\sin\theta = \cos\,(90° - \theta)$;

$\sin\,(90° - \theta) = \cos\theta$

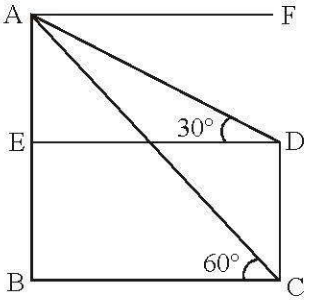
$\therefore \sin 85° = \sin\,(90° - 5°) = \cos 5°$

$\therefore (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + ...$

to terms $+ \sin^2 45° + \sin^2 90°$

$= 8 \times 1 + \dfrac{1}{2} + 1 = 9\dfrac{1}{2}$

17. (b)



AB = 108 m

CD = x metre

From ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3}\ \frac{108}{BC}$

$\Rightarrow BC = \frac{108}{\sqrt{3}} = 36\sqrt{3}$ m

From ΔAED,

$\tan 30° = \frac{AE}{ED}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{108 - x}{36\sqrt{3}}$

$\Rightarrow 108 - x = 36$

$\Rightarrow x = 108 - 36 = 72$ m

18. (c) $\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$

$\Rightarrow \tan 2\theta = \tan(90° - 4\theta)$

$\Rightarrow 2\theta = 90° - 4\theta$

$\Rightarrow 6\theta = 90° \Rightarrow \theta = 15°$

$\therefore \tan 3\theta = \tan 45° = 1$

19. (c) $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

$2\sin\theta + \cos\theta = x$ (Let)

$4\cos^2\theta + \sin^2\theta - 4\sin\theta \cdot \cos\theta + 4\sin^2\theta + \cos^2\theta + 4\sin\theta \cdot \cos\theta$

$= \frac{1}{2} + x^2$

$\Rightarrow 4(\cos^2\theta + \sin^2\theta) + (\cos^2\theta + \sin^2\theta) = \frac{1}{2} + x^2$

$\Rightarrow \frac{1}{2} + x^2 = 5$

$\Rightarrow x^2 = 5 - \frac{1}{2} = \frac{9}{2} \Rightarrow x = \frac{3}{\sqrt{2}}$

20. (c) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow 4\sin\theta = 2\sin\theta \Rightarrow \tan\theta = 2$

$\therefore \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \cos^2\theta(\tan^2\theta - 1)$

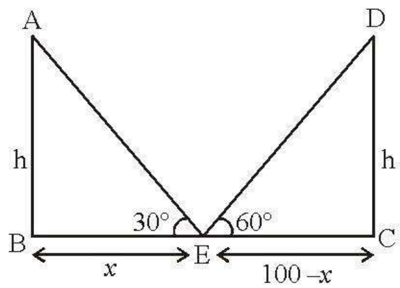
$= \frac{\tan^2\theta - 1}{1 + \tan^2\theta} = \frac{4-1}{1+4} = \frac{3}{5}$

21. (c) $(\tan 1° \cdot \tan 89°) \cdot (\tan 2° \cdot \tan 88°) \ldots\ldots \tan 45°$

$= (\tan 1° \cdot \cot 1°)(\tan 2° \cdot \cot 2°) \ldots. 1$

$= 1 \begin{bmatrix} \because \tan(90° - \theta = \cot\theta \\ \cot(90° - \theta) = \tan\theta, \\ \tan\theta \cdot \cot\theta = 1 \end{bmatrix}$

22. (a)



AB = CD = h metre (Height of pole)

From ΔABE,

$$\tan 30° = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x \quad ....(i)$$

From ΔDEC,

$$\tan 60° = \frac{h}{100 - x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{100 - x}$$

$$\Rightarrow \sqrt{3}\,(100 - x)\;\; h$$

$$\Rightarrow \sqrt{3}\,(100 - \sqrt{3}h) = h$$

[From equation (i)]

$$\Rightarrow 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$$

$$\Rightarrow h = 25\sqrt{3} \text{ metre}$$

23. (a) $\sec^2\theta + \tan^2\theta = 7$

$$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$$

$$\Rightarrow 2\tan^2\theta = 7 - 1 = 6$$

$$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

24. (d)

$$\frac{\sin 39°}{\cos 51°} + 2\tan 11°.\tan 79°.\tan 31°.\tan 59°.\tan 45°$$

$$-3(\sin^2 21° + \sin^2 69°)$$

$$= \frac{\sin 39°}{\cos(90° - 39°)} + 2\tan 11°.$$

$$\tan(90° - 11°).\tan 31°.\tan(90° - 59°).$$

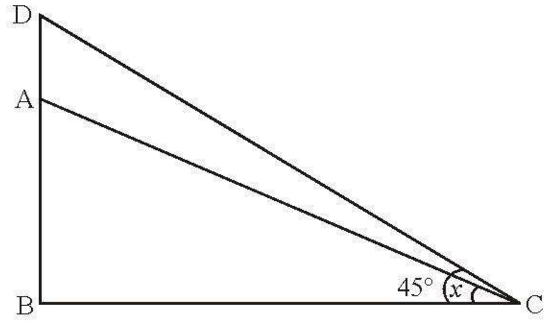
$$1 - 3(\sin^2 21° + \sin^2(90° - 21°)$$

$$= \frac{\sin 39°}{\sin 39°} + 2\tan 11°.\cot 11°.\tan 31°.\cot 31° -$$

$$3(\sin^2 21° + \cos^2 21°) = 1 + 2 - 3 = 0$$

$$[\tan\theta.\cot\theta = 1, \sin^2\theta + \cos^2\theta = 1]$$

25. (b)



AB = Building = h metre

AD = Chimney = y metre

From ΔBCD,

$$\tan 45° = \frac{BD}{BC} \Rightarrow 1 = \frac{h + y}{BC}$$

$$\Rightarrow BC = h + y \quad ......(i)$$

From ΔABC,

$$\tan x = \frac{AB}{BC}$$

$$\Rightarrow \tan x = \frac{h}{BC}$$

$$\Rightarrow BC = h\cot x \quad ......(ii)$$

From equations (i) and (ii), $h + y = h\cot x$

$$\Rightarrow y = (h\cot x - h) \text{ metre}$$

26. (c) $\dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

$$\Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$$

$$\Rightarrow 4\cos^2\theta = 3\cot^2\theta = 3\frac{\cos^2\theta}{\sin^2\theta}$$

$$\Rightarrow 4\cos^2\theta - \frac{3\cos^2\theta}{\sin^2\theta} = 0$$

$\Rightarrow \cos^2\theta\left(4-\frac{3}{\sin^2\theta}\right)=0$

$\therefore 4-\frac{3}{\sin^2\theta}=0$

$\Rightarrow 4\sin^2\theta=3$

$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}=\sin 60°$

$\Rightarrow \theta=60°$

$\cos^2\theta=0 \Rightarrow \theta=90°$

27. (c) $A=\tan 11°.\ \tan 29°$

$B = 2\cot 61°.\ \cot 79°$

$= 2\cot(90°-29°)\cot(90°-11°)$

$= 2\tan 29°.\ \tan 11°$ [$\because \cot(90°-\theta)=\tan\theta$]

$= 2A$

28. (b) $\cos^2\alpha+\cos^2\beta=2$

$\Rightarrow 1-\sin^2\alpha+1-\sin^2\beta=2$

$\Rightarrow \sin^2\alpha+\sin^2\beta=0$

$\Rightarrow \sin\alpha=\sin\beta=0$

$\Rightarrow \alpha=\beta=0$

$\therefore \tan^3\alpha+\sin^5\beta=0$

29. (b) $\tan(2\theta+45°)=\cot 3\theta$

$=\tan(90°=3\theta)$

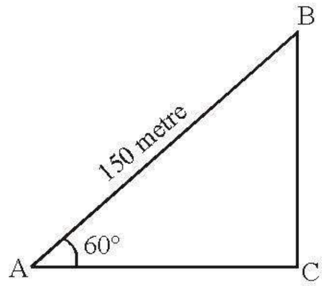
$\Rightarrow 2\theta+45°=90°-3\theta$

$\Rightarrow 5\theta=90°-45°=45°$

$\therefore \theta=9°$

30. (b) AB = Length of the thred = 150 metre

$\angle BAC=60°$



In ΔABC,

$\sin 60°=\frac{BC}{AB}\Rightarrow\frac{\sqrt{3}}{2}=\frac{BC}{150}$

$\Rightarrow BC=150\times\frac{\sqrt{3}}{2}=75\sqrt{3}$ metre

31. (b) $\cos\theta=\frac{15}{17}$

$\Rightarrow \sec\theta=\frac{1}{\cos\theta}=\frac{17}{15}$

$\therefore \cot(90°-\theta)=\tan\theta$

$=\sqrt{\sec^2\theta-1}$

$=\sqrt{\left(\frac{17}{15}\right)^2-1}=\sqrt{\frac{289}{225}-1}$

$=\sqrt{\frac{289-225}{225}}=\sqrt{\frac{64}{225}}=\frac{8}{15}$

32. (a) $\sec^2\theta-\tan^2\theta=1$

$\sec^2\theta+\tan^2\theta=\frac{7}{12}$

$\therefore \sec^4\theta-\tan^4\theta$

$=(\sec^2\theta-\tan^2\theta)(\sec^2\theta+\tan^2\theta)$

33. (a) $\cos x+\cos y=2$

$\because \cos x\le 1$

$\Rightarrow \cos x=1;\ \cos y=1$

$\Rightarrow x=y=0°$

$\therefore \quad \sin x = \sin y = 0$

34. (a) $\tan 15° . \cot 75° + \tan 75° . \cot 15°$

$= \tan 15° . \cot (90° - 15°) + \tan (90° - 15°) . \cot 15°$

$= \tan^2 15° + \cot^2 15°$ ......(i)

$$\begin{bmatrix} \because \tan(90° - \theta) = \cot\theta \\ \cot(90° - \theta) = \tan\theta \end{bmatrix}$$

$\tan 15° = 2 - \sqrt{3}$

$\therefore \quad \cot 15°$

$$= \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$$

$= 2 + \sqrt{3}$

$\therefore \quad \tan^2 15° + \cot^2 15°$

$= (2 - \sqrt{3})^2 + (\sqrt{2} + \sqrt{3})^2$

$= 2\,(4 + 3) = 14$

35. (b) $\tan\theta + \cot\theta = 2$

$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$

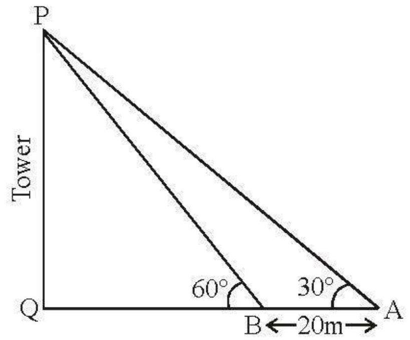
$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \cot\theta = 1$

$\therefore \quad \tan^5\theta + \cot^{10}\theta = 1 + 1 = 2$

36. (c)



Let PQ = h metre and BQ = x metre.

From Δ APQ,

$$\tan 30° = \frac{h}{x+20}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} + \frac{h}{x = 20}$

$\Rightarrow \quad \sqrt{3h} = x + 20$ ....(i)

From Δ PQB,

$$\tan 60° = \frac{PQ}{BQ} = \frac{h}{x}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

$\Rightarrow \quad x = \frac{1}{\sqrt{3}}h$ ....(ii)

$\therefore \quad \sqrt{3h} = \frac{1}{\sqrt{3}}h + 20$

[From equation (i) and (ii)]

$\Rightarrow \quad 3h - h = 20\sqrt{3}$

$\Rightarrow \quad 2h = 20\sqrt{3}$

$\therefore \quad h = 10\sqrt{3}$ metre

37. (a) $\sin\theta - \cos\theta = \frac{7}{13}$ ...(i)

$\sin\theta + \cos\theta = x$ ....(ii)

On squaring both equations and adding,

$2(\sin^2\theta + \cos^2\theta) = \frac{49}{169} + x^2$

$\Rightarrow \quad x^2 = 2 - \frac{49}{169} = \frac{338-49}{169}$

$= \frac{289}{169} \Rightarrow x = \frac{17}{13}$

38. (d) $\sin(2x - 20°) = \cos(2y + 20°)$

$\Rightarrow \quad \sin(2x - 20°) = \sin(90° - 2y - 20°)$

$\Rightarrow \quad 2x - 20° = 70° - 2y$

$\Rightarrow \quad 2x + 2y = 70 + 20 = 90°$

$\Rightarrow \quad x + y = 45°$

$\therefore \quad \tan(x = y) = \tan 45° = 1$

39. (b) Let the number of terms be n, then

By $t_n = a + (n-1)d \; 85 = 5 + (n-1)$

$\Rightarrow \quad n - 1 = 85 - 5 = 80$

$\Rightarrow \quad n = 81$

$\therefore \quad \sin^2 5° + \sin^2 6° + ... + \sin^2 45° + ...$

$+ \sin^2 85° + \sin^2 85°$

$= (\sin^2 5° \;\; \sin^2 85°) + (\sin^2 6° + ... +$

$\sin^2 84°) + ... +$ to 40 terms $+ \sin^2 45°$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 6° + ...$

$+ \cos^2 6°) + ... +$ to 40 terms $+ \sin^2 45°$

$\left[\begin{array}{c} \sin(90° - \theta) = \cos\theta \\ \sin^2\theta + \cos^2\theta = 1 \end{array}\right]$

$= 40 + \frac{1}{2} = 40\frac{1}{2}$

40. (b) $\sin\theta + \text{cosec}\theta = 2$

$\Rightarrow \quad \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \quad \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow \quad (\sin\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = 1 \Rightarrow \text{cosec}\theta = 1$

$\therefore \quad \sin^{100}\theta + \text{cosec}^{100}\theta = 1 + 1 = 2$